

SYSTEMS AND SIGNALS ONLINE QUESTIONS AND GRADING

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Abstract

In this paper we present an E-Learning solution, adapted for online generation of questions for students self study for the new Systems and Signals course. The Systems and Signals course was classically focused on Electrical Engineering but the Berkeley approach radically changed this view and produced a new course that spreads to different areas like Mechanical or Computer Science Engineering. We developed an original framework for this new course that consists of a set of more than 20 generic questions with more than 100 variations each. The students may freely use this framework for self study. For grading, the quizzes are sent to the student's personal area and the grading is immediate and automatic. Results, from other courses where this framework is being used for automatic grading, show that it actually helps the students to follow the course along the semester rather than concentrating all the work on final exams.

1. Introduction

E-Learning tools are growing in popularity and subject of intense researches and funding. Several studies have been carried out to provide e-learning systems for different areas as Algebra [Santos 2002], Electrical Circuits Analysis [Scutaru 2004], Embedded Systems [Gomes 2004], Introductory Courses in Electrical Engineering [Kukk 2004], among many others. At first, most of the e-learning systems were mainly devoted to introductory levels but nowadays we may find e-learning tools for almost all levels.

Systems and Signals classical approach was mainly related to analog mechanical or electrical hardware, which is more related to the origins of the discipline than to its contemporary reality [Lee 2000]. Lee and Varaiya, at Berkeley, introduced a new course on Systems and Signals with a radical different view.

For this new course, Lee and Varaiya developed a set of interactive demos, which is available on the Berkeley course web page. These demos are very useful for

students self study. However, for such a new course there isn't yet any e-learning tool adapted for online generation of questions, correction and students grading.

To fill this gap, we developed a set of robust and tested interactive questions to be introduced in the IST E-Learning System for online generation of midterms and quizzes with automatic correction and students grading.

2. Modern trends for teaching Systems and Signals

Many engineering schools still have introductory courses on Signals and Systems which are mainly related to analog electrical hardware which are more related to the origins of the discipline than to its contemporary reality [Lee 2000].

At Berkeley the electrical Engineering and Computer Science Curriculum was revised to introduce a new course about signals and systems. Lee and Varaiya, revised the curriculum and produced a course designed for Berkeley's 15 week semester as well as an accompanying text book [Lee 2003].

In their book the authors sustain that in the origins, a signal was a voltage that varies over time. Now it is likely to be a sequence of discrete messages sent over the internet using TCP/IP. The state of a system used to be adequately captured by variables in a differential equation. Now it is likely to be the registers and memory of a computer or the state of a set of concurrent finite state machines [Lee 2003].

According to these ideas Lee and Varaiya created a new course that begins by describing signals as functions, focusing on characterizing the domain and the range. Systems are also described as functions, but now the domain and range are sets of signals. Characterizing these functions is the topic of the course. Systems are initially described using the notion of state, first using automata theory and then progressing to linear systems. Frequency domain concepts are introduced as a complementary toolset, different from that of state machines, and much more powerful when applicable. Frequency decomposition of signals is introduced using

psychoacoustics, and gradually developed until all four Fourier transforms (the Fourier series, the Fourier transform, the discrete-time Fourier transform, and the DFT) have been described. Lee and Varaiya linger on the first of these, the Fourier series, since it is conceptually the easiest, and then quickly present the others as simple generalizations of the Fourier series. Finally, the course closes by using these concepts to study sampling and aliasing [Lee 2003], [Lee 2000].

3. E-Learning at IST

LEIA [Pargana 2003] is an E-learning framework developed at Instituto Superior Técnico (IST). This framework is build with symbolic programming (Mathematica) and allows the user to develop a symbolic template question to be instanciated. This template question is coded in a Mathematica Notebook. Individual instances are XML pages generated by this Mathematica Notebook. Template questions are, in general, coded in such a way that when they are processed using LEIA the following steps are executed:

- a) Symbolic parameters, such as the function defining the input of the system or the equation defining the system, are randomly determined. For example, the coefficients of such an equation can be randomly selected from within a certain interval and the input function can be randomly selected from a previously defined list of reasonable input functions.
- b) The solution, for those inputs, is symbolically computed by Mathematica
- c) Different error answers are computed
- d) The XML code for a page containing the text of the question and information about the right and wrong answers is produced.

Different runs of the same Notebook provide different instances of the same question.

The output follows the IMS Question and Test Interoperability v1.2 specification with an extension for supporting pre-generated multiple instances of a single template question.

Mathematica powerful symbolic processing and function library allows for the definition of many complex questions. Such questions are difficult, if not impossible, to code using conventional authoring tools.

The framework is totally integrated in the academic management system Fénix [Cardoso 2003], allowing for automatic the assignment of quizzes to students and corresponding automatic grading. It has been used mainly in Algebra courses [Santos 2002], [Pargana 2003] and more recently in Analysis courses. A significant part of the evaluation is made electronically. The main goal of the work presented here is to take advantage of the successful experience in Algebra and Analysis, and therefore to develop a set of interactive questions for the Systems and Signals course following Berkeley approach. The new template questions use

Mathematica's signal processing package. More than 100 variations are automatically generated, from each template, for each exercise.

4. Subjects and questions

Interactive Questions	
1	State Machines
2	Signals Domain
3	Fourier Series Coefficients
4	Linear time Invariant Systems - definitions
5	Feedback: Fixed point
6	ABCD representation of an ARMA system
7	Frequency Response of Composed Systems
8	Output of an ARMA system
9	Linear time Invariant Systems - definitions (2)
10	State Machines - Definitions
11	Frequency and Zero State Impulse Responses of a feedback system
12	Upsampling/Downsampling
13	Magnitude of the Frequency Response
14	DTFT/CTFT of a discrete/continuous signal
15	Signals periodicity
16	Output of an ARMA system (2)
17	Internal State of an ARMA system
18	Comb Filters
19	Signal Flow Graph equivalent filters

Table 1: List of questions

Questions	Chapters									
	Signals and Systems	Defining Signals and Systems	State Machines	Composing State Machines	Linear Systems	Frequency Domain	Frequency Response	Frequency Response	The Four Fourier Transforms	Sampling and Reconstruction
1			x							
2	x	x								
3									x	
4					x		x			
5				x						
6					x					
7				x			x			
8					x		x			
9					x	x	x			
10		x	x	x						
11				x		x	x			
12										x
13						x	x			
14									x	
15						x				
16					x					
17					x					
18							x	x		
19								x		
Total	1	2	2	4	6	4	7	2	2	1

Table 2 : Matters covered by each question

This work main goal was to take advantage on the successful Maths experience and develop a set of interactive questions, for the Systems and Signals course, following Berkeley approach.

The developed questions were made according to the subjects covered by Lee and Varaiya's text book [Lee 2003]. The questions covered all the book chapters. A list of most of the developed questions is described in Table 1.

The distribution of the questions subjects among the book chapters is presented in Table 2.

As we may see, all book chapters are covered.

Linear systems and frequency response were the most covered chapters not only because of their importance but also because the Mathematica and its signal processing toolbox were well suited to handle these chapters. With the appropriate formulas and analysis we are able to mix in the same question discrete and continuous time systems

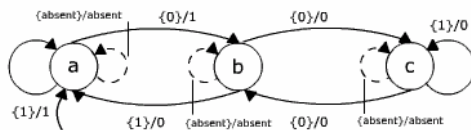
It would be tedious to present here all the developed questions so, in next subsections we briefly present some of the most relevant developed questions.

4.1. State machines

We start with simple and very intuitive question. This is a simple question that exploits the students' knowledge about machine states structure. Figure 1 shows one sample of a state machines question. It is interactively presented to the student for self learning.

The student may interactively answer to the question and check the answer. The questions are available at a web page letting the student make all the retries he wants during his self study.

For the following State Machine



The Initial State is

- A {1,0,absent}
- B {a,b,c} ❌
- C a ✅
- D b

Correct New Question

Figure 1: State Machines Structure

Consider an LTI system with input $x(t) = e^{i\omega t}$

Moreover, consider that the system is described by the following equation:

$$\forall_t \in \text{Reals}, -13y(t) + 2y'(t) - 3y''(t) - 14y^{(3)}(t) + 9y^{(4)}(t) = -5x(t) + 10x'(t) + x^{(3)}(t) - 10x^{(4)}(t)$$

The output response is given by:

- A $y(t) = e^{i\omega t} \frac{9\omega^4 + 3i\omega^3 + 14\omega^2 + 2i\omega - 5}{-10\omega^4 - 10\omega^2 + i\omega - 5}$
- B $y(t) = e^{i\omega t} \frac{9\omega^4 + 14i\omega^3 + 3\omega^2 + 2i\omega - 13}{-10\omega^4 - i\omega^3 - 10\omega^2 - 5}$
- C $y(t) = \frac{-10\omega^4 - i\omega^3 - 10\omega^2 - 5}{9\omega^4 + 14i\omega^3 + 3\omega^2 + 2i\omega - 13}$
- D $y(t) = e^{i\omega t} \frac{-10\omega^4 - i\omega^3 - 10\omega^2 - 5}{9\omega^4 + 14i\omega^3 + 3\omega^2 + 2i\omega - 13}$
- E $y(t) = e^{i\omega t} \frac{-13 + 2e^{-i\omega} - 3e^{-2i\omega} - 14e^{-3i\omega} + 9e^{-4i\omega}}{-5 + 10e^{-2i\omega} + e^{-3i\omega} - 10e^{-4i\omega}}$
- F $y(t) = e^{i\omega t} \frac{-5 + 10e^{-2i\omega} + e^{-3i\omega} - 10e^{-4i\omega}}{-13 + 2e^{-i\omega} - 3e^{-2i\omega} - 14e^{-3i\omega} + 9e^{-4i\omega}}$

Correct New Question

Figure 2: Output of an ARMA system (continuous)

When clicking over "New Question", the student is presented several variations of the question which include new state machines diagrams and new questions about. The question may be on the inputs, outputs, states or initial state of the state machine represented in the diagram.

When the student is being evaluated, the new question option does not exist and is replaced by a submit button

Consider an LTI system with input $x(n) = e^{i\omega n}$

Moreover, consider that the system is described by the following equation:

$$\forall_n \in \text{Integers}, -4y(n-4) + 14y(n-3) - y(n-2) - 10y(n-1) + 3y(n) = 11x(n-4) + 12x(n-3) - 11x(n-2) + 13x(n-1) - 7x(n)$$

The output response is given by:

- A $y(n) = e^{i\omega n} \frac{-7 - 10e^{-i\omega} + 14e^{-2i\omega} - e^{-3i\omega} - 4e^{-4i\omega}}{3 + 12e^{-i\omega} - 11e^{-2i\omega} + 13e^{-3i\omega} + 11e^{-4i\omega}}$
- B $y(n) = e^{i\omega n} \frac{11\omega^4 - 12i\omega^3 + 11\omega^2 + 13i\omega - 7}{-4\omega^4 - 14i\omega^3 + \omega^2 - 10i\omega + 3}$
- C $y(n) = e^{i\omega n} \frac{3 - 10e^{-i\omega} - e^{-2i\omega} + 14e^{-3i\omega} - 4e^{-4i\omega}}{-7 + 13e^{-i\omega} - 11e^{-2i\omega} + 12e^{-3i\omega} + 11e^{-4i\omega}}$
- D $y(n) = e^{i\omega n} \frac{-7 + 13e^{-i\omega} - 11e^{-2i\omega} + 12e^{-3i\omega} + 11e^{-4i\omega}}{3 - 10e^{-i\omega} - e^{-2i\omega} + 14e^{-3i\omega} - 4e^{-4i\omega}}$
- E $y(n) = e^{i\omega n} \frac{-4\omega^4 - 14i\omega^3 + \omega^2 - 10i\omega + 3}{11\omega^4 - 12i\omega^3 + 11\omega^2 + 13i\omega - 7}$
- F $y(n) = \frac{-7 + 13e^{-i\omega} - 11e^{-2i\omega} + 12e^{-3i\omega} + 11e^{-4i\omega}}{3 - 10e^{-i\omega} - e^{-2i\omega} + 14e^{-3i\omega} - 4e^{-4i\omega}}$

Correct New Question

Figure 3: Output of an ARMA system (discrete)

and the student is automatically graded.

4.2. Output of an ARMA system

This is a more demanding question which exploits the computing of the output of a LTI system (figure 2)

The LTI system is given by its state equation with the Auto Regressive (AR) and Moving Average (MA) coefficients.

As the input is a pure complex exponential and this is a LTI system, the student will mainly need to evaluate the Frequency Response and multiply it by the input to obtain the output of this system.

The Frequency response will be obtained replacing $x(n)$ by $e^{j\omega n}$ and $y(n)$ by $H(\omega) e^{j\omega n}$.

Solving the equation for $H(\omega)$ the AR coefficients will stay on the numerator and the MA coefficients will go to the denominator.

All the $e^{j\omega n}$ terms will disappear and the student will get the frequency response.

A lot of variations exist for this question. The number of coefficients in the AR or the MA part varies and automatically Mathematica computes the right and false responses.

Moreover and much more interesting, the system might be represented as a difference equation instead of a differential equation.

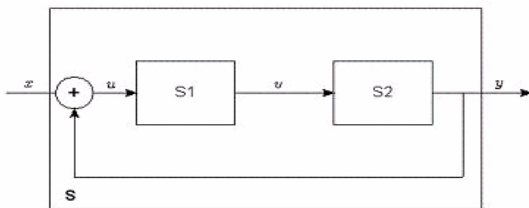
Hence, this question is general enough to include continuous and discrete systems. Figure presents a discrete system version of this question.

The calculations and techniques necessary for solving discrete or continuous system version of this question are much different. However, the generic question is the same. As we see, the variations for each question may involve different subjects and chapters, rather than just numerical variations of a given formula.

4.3. Frequency Response of composed systems

This question exploits the several ways how systems can be combined and how this affects their frequency response.

Consider the following composite system:



Subsystems S1 and S2 are defined by the following equations:

$$\forall t \in \text{Reals}, -4v(t) = -8u''(t)$$

and

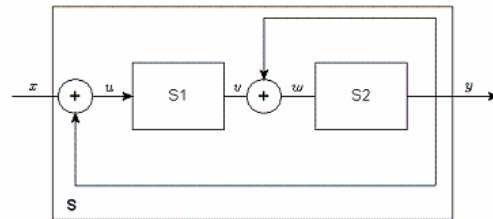
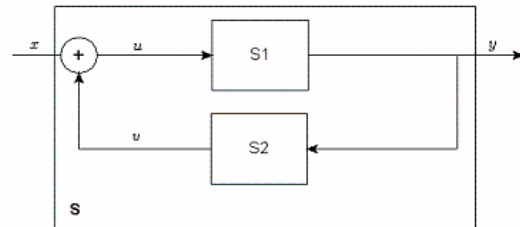
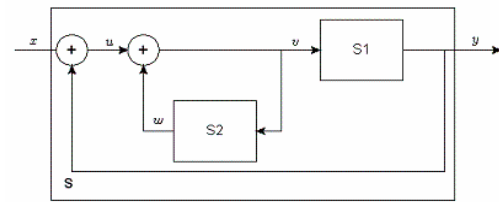
$$\forall t \in \text{Reals}, 6y(t) - 9y^{(8)}(t) = -3v(t)$$

The S frequency response is:

- A $H(\omega) = \frac{-(2\omega^2+9)(3\omega^8-2)}{1}$
- B $H(\omega) = \frac{3\omega^8-2\pi\omega^2-2}{6\omega^{16}+3\omega^8-4\omega^2-1}$
- C $H(\omega) = \frac{-2\omega^2}{3\omega^8+2\omega^2-3}$
- D $H(\omega) = \frac{-2\omega^2(3\omega^8-2)}{3(8\omega^8-5)}$
- E $H(\omega) = \frac{-2\omega^2}{3\omega^8+2\omega^2-2}$
- F $H(\omega) = \frac{3\omega^8-2\omega^2-2}{3\omega^8+2\omega^2-1}$

Correct

New Question



Systems can be combined in several forms: side-by-side, cascade, product form and feedback [Lee 2003]. Cascade and feedback system's composition and their

Figure 4: Frequency response of composed systems

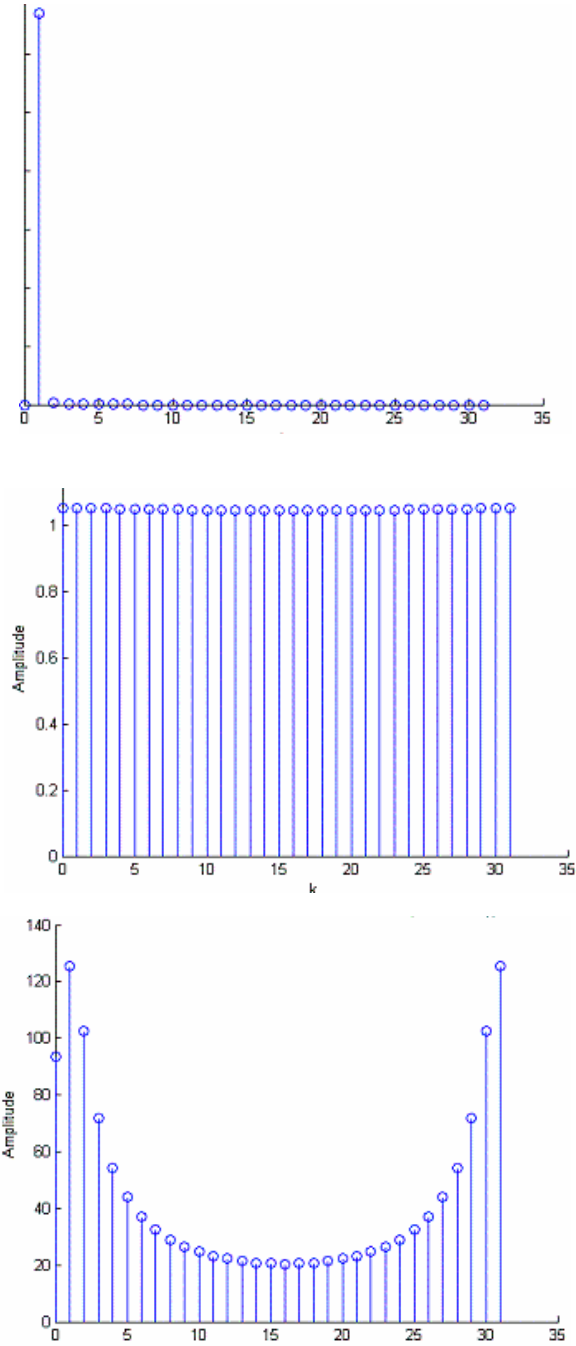


Figure 6: Top: discrete signal; Middle: right answer; Bottom: wrong answer

frequency response are the subject of the question illustrated in figure 4.

To solve this question, the student will find the frequency response of each subsystem, combine them according to the composition (in this case, a concatenation and a feedback) and calculate the final

frequency response.

Like other questions, a lot of variations exist for this question: the systems may be continuous or discrete and there are a lot of topologies for the system composition. Figure 5 presents some implemented alternatives.

4.4. Identification of the Fourier series coefficients

This question exploits the ability of the student for finding the Fourier Series coefficients for a periodic signal.

The question consists on the plot of a signal and a set of plots for the Fourier Series coefficients. The student has to identify which plot of the coefficients matches the plot of the signal.

The main originality of this question is that it is almost useless for the student to know the formula for the calculus of each coefficient because the signals are plotted instead of giving an analytical expression for the signal.

The main goal is that the students actually understand the meaning of each coefficient. For instance, the student will always identify the right plot if he understands the following:

- The first coefficient will represent the signal average value.
- A Kronecker delta will have components in all frequencies.
- A pure sinusoid will only have one coefficient.
- A signal with a lot of variation will have components in higher frequencies.

With these simple rules we check if the student actually understands the meaning of the Fourier series, rather than blindly apply the formulas.

Figure presents some plots of this question.

The plot on the top shows a discrete signal which is just a shifted Kronecker Delta impulse.

The student must be able to understand that the phase shift will not have any impact on the amplitude of the Fourier Series coefficients and, as it is a pure Kronecker Delta impulse, it would have components in all the frequencies.

Hence the middle plot is the right answer and not the bottom plot (see figure 6, or the interactive questions web page for a clearer view).

5. Supplements and Students Feedback.

The set of questions involves much more subjects like filter design, sampling and aliasing, fundamental concepts or Linear Invariant Systems. Being tedious to prolong the details of the other questions, the main relevant information is that all the chapters are covered

and the questions are much more generic than simple numerical variations.

The whole set of interactive questions is available at http://mega.ist.utl.pt/~smcos/sistemas_sinais/.

Some questions involve some non trivial calculations. To help the students self learning several questions, we solved in detail some questions from the final exams, which, in turn, had a strong part based on the available interactive questions.

The set of solved exercises is being object of a supplement to the text book which is in preparation. Together with the text book supplement, a companion CD will allow the students the installation of the software with the interactive questions for self study.

The set of question was left under test for students and the academic community. To have an idea of the impact on the students the page web counter registered more than 1300 access during the two months period from the end of December to the end of February. Taking into account that only access from different IP addresses where counted, and knowing that most internal accesses through DHCP present the same router IP address and hence are not counted, this number is just a lower bound of the effective number of accesses.

In the students final report they state that this set of interactive questions as well as the solved exams exercises were of good help to achieve success in this course.

We opened a dedicated thread in the student's forum. The students intensively used this forum for testing the exercises, correct minor mistakes and bugs, and get some advice about the way of solving the exercises (there was a small prize for each bug detected).

The forum registered 151 posts in the period from end of December till the end of February. The posts were important to correct minor bugs. The set of questions is now tested and may be introduced on the IST E-learning platform.

6. Conclusions and further work

We presented a set of interactive and questions for self studying and automatic grading for students of a Systems and Signals course.

The set of questions covers all the chapters of the reference book [Lee 2003]. The initial number of questions is 22 but they are very general, much more than just numerical variations of the same problem. Nevertheless this was just the first year. This number will increase during next years to achieve a larger database.

The students showed a lot of interest on these interactive questions which can be monitored by the number of accesses to the page (more than 1300) and the number of posts on the forum (151). It is a good score

considering that there are more or less 100 students and the questions were only made available by the end of the semester.

The forum was useful to correct minor bugs on the questions. The set of questions is now much more error free, robust and trustful to be introduced on the E-Learning platform.

The goal is that next year the variations of the questions will be sent to the students' area along the semester.

The questions will be automatically corrected and the students will be immediately graded. The questions will be available for all the students at the same time for a short period to avoid cheating.

We expect that this procedure will make the students, instead of concentrating all the effort in the last weeks of the semester, to follow the course along the semester as it happened with the IST courses that are already using this platform.

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