

Design of Optimal Hygrothermally Stable Laminates with Extension-Twist Coupling by Ant Colony Optimization

Aditya P. Apte¹, Robert A. Haynes², Bo Ping Wang³, Erian A. Armanios⁴

¹ Dept. of Radiation Oncology, Washington University School of Medicine, St. Louis, USA, aditya.apte@gmail.com

² Dept. of Aerospace Engineering, Georgia Institute of Technology, Atlanta, USA, robert.haynes@gatech.edu

³ Dept. of Mechanical and Aerospace Engineering, The University of Texas, Arlington, USA, bpwang@uta.edu

⁴ Dept. of Mechanical and Aerospace Engineering, The University of Texas, Arlington, USA, armanios@uta.edu

Abstract

Laminated composite materials allow for tailoring of their elastic properties through deliberate selection of the orientation of their constituent laminas (referred to as ply angles). Among the achievable properties is a coupling between in-plane and out-of-plane deformation modes. Extension-Twist is a specific type of coupling which has applications in rotor blades to change the angle of attack with a change in rotor speed. The optimal Ply-angles for E-T coupling have been obtained in previous work using Sequential Quadratic Programming. Since the Objective and Constraints are highly nonlinear, there exist multiple local and global solutions. This work tried to explore through the use of Ant Colony Optimization (ACO), a global optimizer, whether there are any better designs than those obtained in previously work. It was verified that the E-T couplings obtained in previous work was global optimal. In addition, ACO produced new global optimal designs which were similar to the previous designs in terms of objective and constraint satisfaction.

Keywords: Ant Colony Global Composite Coupling

1. Introduction

Coupling of deformation modes can be used advantageously to elastically tailor the response of a composite structure. For example, extension-twist coupling has applications ranging from wind turbine blades to tilt-rotor aircraft. Other types of coupling include extension-bending, bending-twist, and anticlastic coupling. Extension-twist coupling is only achievable using asymmetric stacking sequences, which often produce hygrothermal instabilities, meaning temperature or moisture changes cause bending or warping [1]. Past design methods involve obtaining the desired coupling properties then attempting to minimize hygrothermal effects [2]. Sequential Quadratic Programming (SQP) has previously been used to satisfy hygrothermal stability as the primary constraint then investigates the achievable elastic properties.

The first established family of extension-twist-coupled composites that retain hygrothermal stability was detailed by Winckler [3] and given by the stacking sequence

$$[\theta / (\theta-90)_2 / \theta / -\theta / (90-\theta)_2 / -\theta]. \quad (1)$$

These Winckler-type laminates have been the industry standard since their introduction. A review of their usage has been published previously [4]. It has been discovered, however, that this family does not contain all hygrothermally stable laminates. Further, it has also been found that Winckler's family does not maximize extension-twist coupling among laminates consisting of eight plies. This work uses a global optimizer, Ant Colony Optimization to explore better designs with hygrothermal stability enforced as given in previous work [5]. Ant Colony Optimization is described followed by optimization problem formulation and results.

2. Ant Colony Optimization

In the last decade various algorithms have been researched to solve optimization problems based on prey catching behavior of ants. Such algorithms find application in robotics, objects clustering, communication network and combinatorial optimization. The ants whose behavior was modeled in this work are found in Mexican desert and are known as *Pachycondyla apicalis* [8]. These ants use visual cues as feedback mechanism for catching preys. They use simple principles to search their preys, both from global and local viewpoint. Starting from their nest, they globally cover a given surface by partitioning it into many hunting sites. Each ant performs a local random exploration of its hunting sites and choice of site is sensitive to the success previously met on this site. These principles have been translated into an algorithm to obtain global optimal solution of unconstrained and constrained optimization problems [8, 9]. Following is a brief algorithm based on the behavior of these ants.

- (1) **Choose** randomly the initial nest location N . This is the starting point to the algorithm.
- (2) For each ant a_i , $i \in [1 \dots n]$:
 - If a_i has less than p hunting sites in memory, then **Create** a new site in the neighborhood of N and explore the created site
 - Else
 - If the previous site exploration was successful then **Explore** again the same site
 - Else **Explore** a randomly selected site (among the p sites in memory)
- (3) Remove from the ants memories all sites which have been explored unsuccessfully more than $P_{local}(a_i)$ consecutive times.
- (4) Perform recruitment (best site copying between two randomly selected ants)
- (5) If more than T iterations have been performed Then **Change** the nest location and **Reset** memories of all ants
- (6) Go to (2) or Stop if a stopping criteria is satisfied

Figure 1 illustrates the global and local hunting strategy. The parameters which govern the spread of global and local sites are A_{site} and A_{local} . Based on the actual behavior of ants these parameters are set such that $A_{site}/A_{local} = 10$. Figure 2 illustrate the convergence of algorithm by moving nest to more interesting areas of the design space. Readers are encouraged to refer to [8] for in depth information about recruitment process and the performance of algorithm. Lagrange multiplier approach was used to solve the constrained optimization problem.

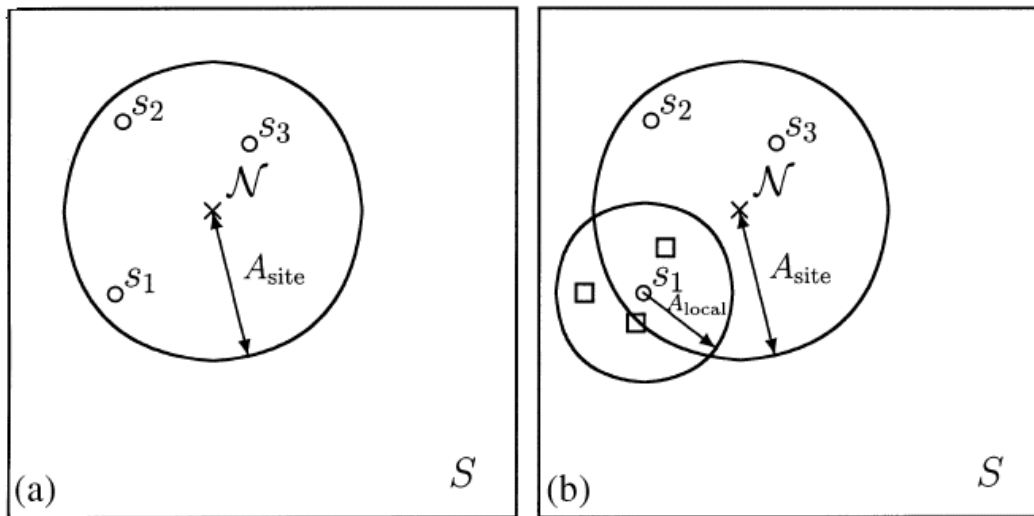


Figure 1 Illustration of Global and Local search mechanism: (a) Sites s_1 , s_2 and s_3 are randomly generated (their maximum distance from the nest N is given by A_{site}), (b) Squares represent local explorations of site s_1 at a maximum distance A_{local} from s_1 .

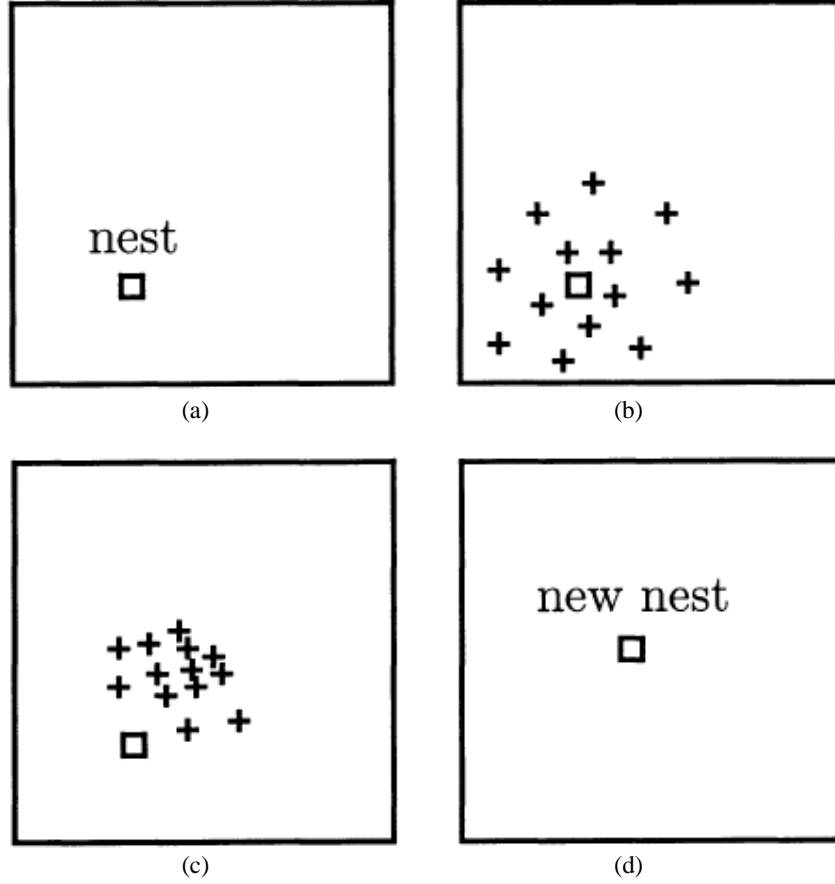


Figure 2 Illustration of algorithm. In (a), the nest is randomly placed in the search space. Then, in (b), hunting sites are randomly created around the nest with the distribution generated by the *Asite* parameters. In (c), due to the local explorations, hunting sites move towards more interesting areas of the search space (here the center of the space in our example). Then, in (d), the nest moves to the best generated point so far. Hunting sites are then created again as in (b), and so on.

3. Optimization Problem Formulation

As given in classical lamination theory [6] for specially orthotropic plies, a laminate's non-mechanical in-plane forces and shears and out-of-plane moments and curvatures are related to the in-plane strains and out-of-plane curvatures as

$$\begin{Bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \\ M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix}^{(T,H)} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ \hline B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \\ \kappa_{xx} \\ \kappa_{yy} \\ \kappa_{xy} \end{Bmatrix} \quad (2)$$

Since hygrothermal stability can be defined as having the out-of-plane curvatures equal to zero for any change in temperature or moisture, this can be expressed as

$$\begin{Bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \\ M_{xx} \\ -M_{xx} \\ M_{xy} \end{Bmatrix}^{(T,H)} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \\ B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} \quad (3)$$

where A_{ij} and B_{ij} , are the in-plane and coupling stiffness coefficients respectively and $()^{(T,H)}$ indicates non-mechanical quantities. The non-mechanical in-plane forces and shears and out-of-plane moments and curvatures for an n -ply laminate are given by

$$\begin{Bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \end{Bmatrix}^{(T,H)} = T_2 \begin{Bmatrix} 1 \\ 1 \\ 0 \end{Bmatrix} + T_3 \sum_{k=1}^n \begin{Bmatrix} \cos 2\theta_k \\ -\cos 2\theta_k \\ \sin 2\theta_k \end{Bmatrix}$$

$$\begin{Bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix}^{(T,H)} = T_1 \sum_{k=1}^n \begin{Bmatrix} \cos 2\theta_k \\ -\cos 2\theta_k \\ \sin 2\theta_k \end{Bmatrix} (2k - n - 1) \quad (4)$$

where θ is the angle of the k^{th} ply, and T_1 , T_2 , and T_3 are solely functions of the material properties and temperature and moisture changes.

Using (2), it can be proven that the necessary and sufficient conditions to ensure hygrothermal stability are either

$$N_{xx}^{(T,H)} = N_{yy}^{(T,H)} \text{ and } N_{xy}^{(T,H)} = M_{xx}^{(T,H)} = M_{yy}^{(T,H)} = M_{xy}^{(T,H)} = 0$$

or

$$(5a, b)$$

$$B_{ij} = 0.$$

This leads to the following objective and constraints for our problem.

Objective is to maximize Extension-Twist term:

$$g(\{\theta_i : i = 1 \dots N\}) = -\beta_{16}^2 \quad (6)$$

Constraints for Hygrothermal Stability can be expressed as:

$$\sum_{k=1}^n (2k - n - 1) \cos 2\theta_k = 0 \quad (7a)$$

$$\sum_{k=1}^n (2k - n - 1) \sin 2\theta_k = 0 \quad (7b)$$

$$\sum_{k=1}^n \sin 2\theta_k = 0 \quad (7c)$$

$$\sum_{k=1}^n \cos 2\theta_k = 0 \quad (7d)$$

Figures 3 and 4 show contours for objective and constraints. It can be seen that the objective and constraints are highly nonlinear which merit the use of global optimizer like ACO.

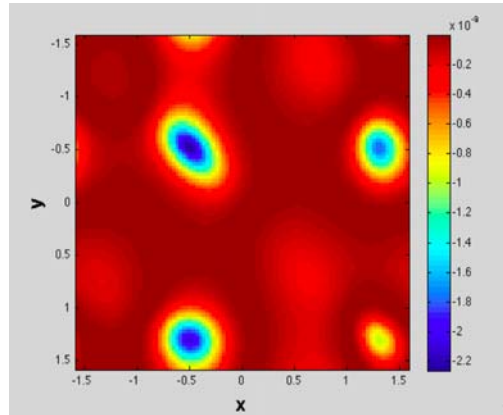


Figure 3 Variation of Objective function (eq. 6) w.r.t. 2nd and 3rd ply angles.

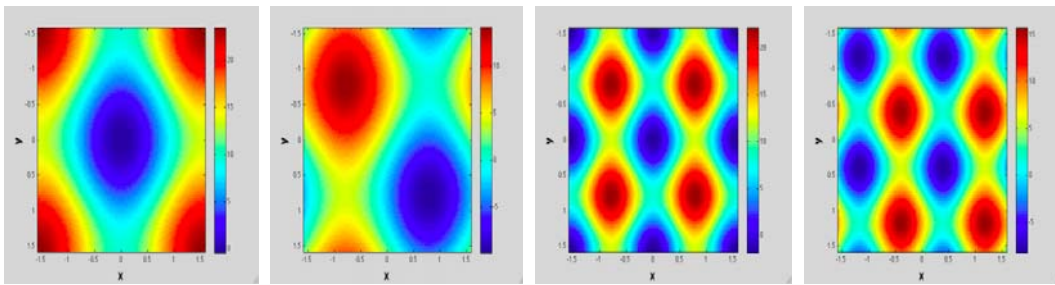


Figure 4 Variation of Constraint functions (eq. 7 a-d) w.r.t. 2nd and 3rd ply angles

4. Results

ACO produced multiple global solutions. It was verified that the results obtained from the previous work were global optimal. These multiple global solutions would be valuable if any other design or manufacturing criteria needs to be imposed further. Table 1 lists the designs from previous work and the ones obtained from the current work.

Table 1: E-T coupling designs from previous and current work

Laminate	Optimal Stacking Sequence (deg): Previous work	Optimal Stacking Sequence (deg): More designs from ACO
5-Ply	[-58.7 / 11.4 / 45 / 78.6 / -31.3]	No new design. ACO produced same solution as previous work
6-Ply	[21.2 / -63.8 / -48.7 / 48.7 / 63.8 / -21.2]	[- 66.95 / 28.98 / 41.79 / -38.61 / -27.74 / 68.73]
8-Ply	[21.5 / -72.1 / -57.9 / 29.6 / -29.6 / 57.9 / 72.1 / -21.5]	[73.67 / -24.43 / 43.87 / -39.22 / -49.77 / 29.09 / 33.55 / -71.88] [-75.11 / 27.71 / 30.21 / -53.37 / -40.31 / 43.92 / -22.61 / 73.41]
10-Ply	[16.2 / -69.0 / -65.3 / 31.8 / 42.1 / -42.1 / -31.8 / 65.3 / 69.0 / -16.2]	[15.09 / -69.06 / 24.27 / -60.54 / -51.82 / 53.81 / 61.28 / -22.38 / 70.51 / -14.29]

In order to compare the new designs with the previous ones, robustness of the all the designs was computed by varying ply-angles from -2 to +2 degrees. Figure 5 shows histograms for objective functions (of previous and new solutions) with the designs perturbed by 2 degrees around the optimal for 8-ply design. Figure 6 shows histograms for sum of squared constraint functions (of previous and new solutions) with the designs perturbed by 2 degrees around the optimal for 8-ply design. It can be observed that both the solutions are almost equally robust – one does better with objective and the other with constraints.

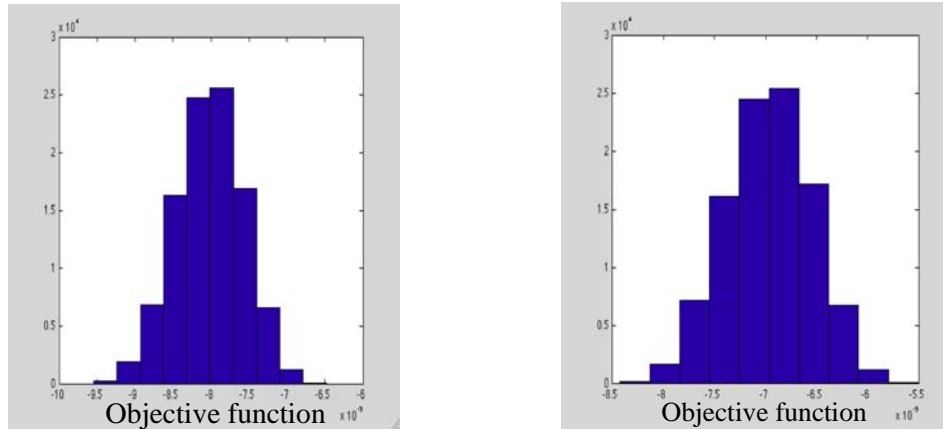


Figure 5 Robustness of Objective. Previous work (Left) and Current work (right). Y-axis represents counts

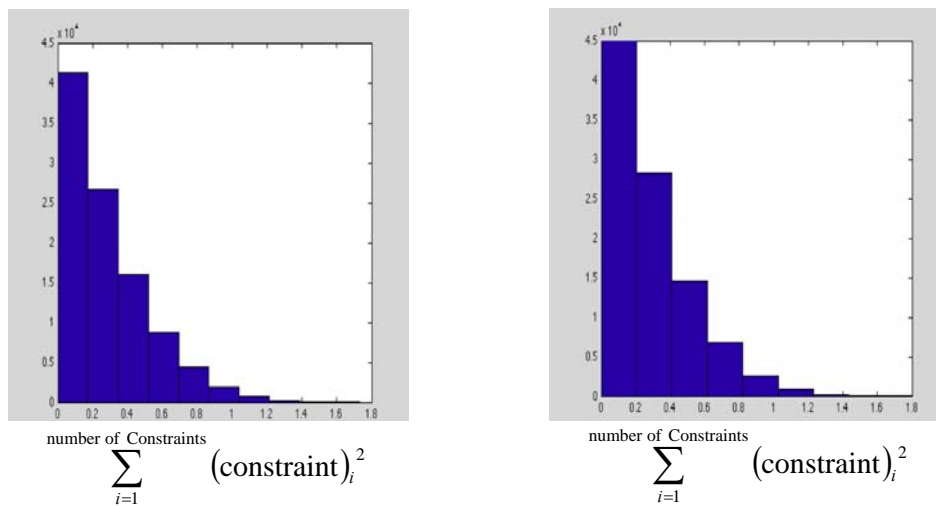


Figure 6 Robustness of Constraints. Previous work (Left) and Current work (right). Y-axis represents counts

5. Conclusion

- 1>It was verified that the E-T couplings obtained in previous work were global optimal.
- 2> In addition, ACO produced new global optimal designs which were similar to the previous designs in terms of objective and constraint satisfaction.

References

- [1] Hyer, M.W., “Some Observations on the Curved Shape of Thin Unsymmetric Laminates,” *Journal of Composite Materials*, Vol. 15, (2), 1981, pp. 175–194.
- [2] Armanios, E. A., Hooke, D., Kamat, M., Palmer, D. and Li, J., “Design and Testing of Composite Laminates with Optimum Extension-twist Coupling,” *Composite Materials: Testing and Design*, edited by E.T. Camponeschi Jr, Vol. 11, American Society for Testing and Materials, Philadelphia, PA, 1993, pp. 249–262.

- [3] Winckler, S. I., "Hygrothermally Curvature Stable Laminates with Tension-torsion Coupling," *Journal of the American Helicopter Society*, Vol. 31, (7), 1985.
- [4] Haynes, R., Carey, R., and Armanios, E.A., "A New Class of Hygrothermally Stable Laminates with Extension-twist Coupling," in *Proceedings of the AHS 65th Annual Forum*, Grapevine, TX, USA, May, 2009.
- [5] Cross, R.J., Haynes, R.A., and Armanios, E.A., "Families of Hygrothermally Stable Asymmetric Laminated Composites," *Journal of Composite Materials*, Vol. 42, (7), 2008, pp. 697-716.
- [6] Jones, R. M., *Mechanics of Composite Materials*, 2nd Ed., Taylor & Francis, Philadelphia, PA, 1999, pp. 190-203.
- [7] Armanios, E. A., Makeev, A., and Hooke, D., "Finite-Displacement Analysis of Laminated Composite Strips with Extension-Twist Coupling," *Journal of Aerospace Engineering*, Vol. 9, (3), 1996.
- [8] N Monmarche, G Venturini, M Slimane, "On how Pachycondyla apicalis ants suggest a new search algorithm", *Future Generation Computer Systems* 16 (2000) 937-946.
- [9] A. Apte and B. P. Wang, "Using power of Ants for Optimization", AIAA-2004-1681, 45th AIAA/ ASME/ ASCE/ AHS/ ASC Structures, Structural Dynamics and Materials Conference, Palm Springs, California, Apr. 19-22, 2004.