Particle Swarm Optimization and the Efficiency Frontier for the Problem of Expansion of an Electrical Network

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Abstract

The increasing demand for electricity implicates in frequent expansion of distribution systems. It is necessary a long-term planning that respects the dynamics of the evolution of demand and the availability of financial resources. Planning is to determine the necessary investments and what the proper configuration of the network in each stage of the planning horizon to achieve the goals. The definition of multiple objectives features intended to meet the needs of both dealers and consumers, in addition, the multiobjective approach can identify the efficient frontier that represents a collection of trade-offs between the objectives set out, among which the decision maker may choose to play the one that best suits their priorities.

This paper presents a model of multi-objective mixed integer programming (MIP), which considers two conflicting objective functions, related costs and lost power for the expansion planning of distribution networks for electricity. Technical and operational constraints are imposed so that the solutions meet the specific characteristics of electrical networks. The model proposed in this work was inspired by Haffner, et al (2008). Some adjustments were made to allow multi-criteria approach to the problem and others in order to adapt it to linear inherent power grids. To overcome this difficulty, was used Particle Swarm Optimization (PSO) to find solutions good enough for the problem under discussion. Since the decision variables of the model are binary, to allow the efficient resolution of even using PSO, we defined a way to assign continuous variables to binary variables where PSO updates are applied. The resolution of the model using the PSO adapted to the problem of expansion and associated with the technique presented in the study allowed the creation of multi-stage planning for the expansion of networks larger than those handled by exact methodology, resulting in a successful approach to the frontier of efficiency.

Key words: Expansion of electrical network, MIP, PSO.

1. Introduction

The increasing demand for electricity necessitates the frequent expansion of distribution systems. This expansion aims to enable the system to maintain, over time, the quality of attendance of existing demand and be able to attend new customers. Since it involves constructions that require large investments and also take time in implementation, it is necessary a long-term planning that respects the dynamic characteristic of the evolution of demand and the availability of financial resources.

According to Vaziri [17] the first article about optimization in distribution planning is attributed to Knight, published in 1960. After that, numerous publications appeared presenting different models and techniques for solving the problem. A model for expansion planning of distribution by Mixed Integer Linear Programming (PPLIM) is proposed in [2]. The papers [10, 11] extend this approach to multi-period formulation. Algorithms branch-and-bound for PPLIM are used in [1,8,9,15,16]. A convex linearization of the cost function is employed in [6] providing thus a continuous model for the problem.

The size of real cases and non-linear characteristics of electric power networks add complexity to the problem of expansion. To overcome this difficulty, some authors make use of metaheuristics in finding sufficiently good solutions to the problem under discussion. The use of simulated annealing is proposed [12] to solve the planning problem. The authors also make an analysis on the influence of algorithm parameters and the size of the network in the results. Ant Colony is another metaheuristic that has been applied to solve the problem in [7].

Energy planning is one of the most active and exciting application of multicriteria methodologies [5]. While costs continue to be the greatest motivation, new requirements for reliability, service quality and security of supply, especially considering the current trends towards the liberalization of the electricity market in Brazil. Thus, in the moment of decision, multiple conflicting objectives must be taken into consideration. Some studies
have been developed using multicriteria methods. We can mention [13] that add to the Simulated Annealing, fuzzy sets, to capture the uncertainties that exist at the time of planning and proposes a multicriteria planning. An evolutionary algorithm to obtain solutions that optimize cost and reliability of the expansion of the network is presented in [14].

The evolution of this project has been done in a direction, regardless of the methodology selected, in order to solve the problem in a multistage. Thus the planner would know the best configuration for the network and the amount of resources needed at each stage. It is hoped with this, to avoid unnecessary work carried out at intermediate stages.

The choice of network configuration should take into account specific features such as radial configuration, maximum capacity of conductors and substations, limits of nodal voltages. Moreover, flows in the branches and the nodal voltages must satisfy the two Kirchhoff's laws.

For purposes of modeling the planning problem, a network of primary power distribution can be regarded as a directed graph composed of n nodes and k branches. Each node represents or a distribution substation (power supply) or a concentration point of load (demand node). The branches represent the feeders that perform the connection between nodes and make the transportation of electricity.

The expansion planning considers a primary existing distribution network operating in good conditions and forecasts the increased load or appearance of new points of demand in different periods of the planning horizon. Assumes prior knowledge of the branches that can be added to network with their choices of gauge and the definition of what substations we can to install.

The model proposed in this paper was presented inspired by [8]. Some adjustments were made for a multicriteria approach to the problem and others in order to adapt it to operate in a Brazilian network. The model also associates to each possible branch of the network, two-way flow path, resulting in a duplication of branches. Furthermore, the model considers, in any of the stages of the horizon as possible actions the expansion of existing substations or the installation of new substations. In the branches the allowed actions in each stage are the installation, alteration and removal of them with a choice of different size. The distribution substations and branches of the network will be treated by facilities. Every possible action is associated with an investment cost which represents the amount of financial resources sufficient to carry out the action that make possible the use of the facility in the time required. Each facility is assigned a cost of operation/maintenance, which represents the amount of financial resources sufficient to maintain the functioning and in good condition, in each period.

It is of interest to the electricity distribution companies that this task is performed at the lowest possible cost. The planning is done to minimize the costs of investments in network expansion and the costs of operation/maintenance of distribution network in all stages of operation.

Although the costs involved in expanding a distribution system continue to be the main concern of the utilities, the changes that have happened in the global energy scenario and in particular the restructuring of the sector in Brazil, require that other concerns are present at the planning. The amount of energy lost in distribution can be considered as an indicator of quality. Thus, we chose to perform the planning also aimed at reducing technical losses in distribution. To find a compromise between the two objectives which are clearly conflicting, we opted for multicriteria approach. Thus, the planner will have several solutions among which it may choose to implement one that best suits their preferences and priorities in the moment of decision. When choosing the planner should be aware that each of the solutions is non-dominated, that one is strictly more efficient than other solutions with respect to one of the criteria necessarily provide lower-quality results for the other criterion.

As a response we wish to know what the investment required and what the proper configuration of the network at each stage to achieve the proposed objectives. The model combines these decisions with binary variables that determine whether the facilities, with their respective characteristics shall be selected for use in the various stages and also what investments would be handled.

2. The exact model
A mixed integer linear programming model was developed to solve the problem of expansion. In the model it was considered the notation presented in Table 1.

<table>
<thead>
<tr>
<th>Item</th>
<th>Meaning</th>
<th>Unity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{op}$</td>
<td>resistance/unit length of the branch k on option op</td>
<td>$\Omega$/km</td>
</tr>
<tr>
<td>$l_k$</td>
<td>length of the branch k</td>
<td>km</td>
</tr>
<tr>
<td>$f_{k,op,t}$</td>
<td>current flow through the branch k, installed in the size option op in the period t</td>
<td>A</td>
</tr>
<tr>
<td>$\delta_t$</td>
<td>factor to update cost in period t</td>
<td></td>
</tr>
<tr>
<td>$O_{k,op}$</td>
<td>operation or maintenance cost of the branch k on option op</td>
<td>$$$</td>
</tr>
</tbody>
</table>
The cost function can be decomposed into two parts where the first, custoinv, represents the present value of the amount of investment needed to get the desired configuration in each period of planning horizon and refers to the higher proportion of the values involved in the expansion a distribution network. The second part, which it was called customanut, represents the present value of expenditure on maintenance and operation of the network with configurations planned. Then the cost function may be represented by Eq. (1).

\[
Custo = \sum_t \delta_t \cdot (custoinv + customanut)
\]

\[
custoinv = \sum_k \sum_{op} CIR_{k,op} \cdot l_k \cdot x_{k,op} + \sum_j \sum_{cap} CIS_{j,cap} \cdot W_{j,cap}
\]

\[
customanut = \sum_k \sum_{op} O_{k,op} \cdot y_{k,op} + \sum_j \sum_{cap} OS_{j,cap} \cdot z_{j,cap}
\]

The loss function calculates the amount of energy lost during the distribution process at all stages of the planning horizon. It can be represented by Eq. (2).

\[
Perdas = \sum_t \sum_{k,op} R_{op} \cdot l_k \cdot f_{k,op}^2
\]

The flow in the branches and the nodal voltages are controlled by the Kirchhoff’s laws and can be described by Eq.(3) that inform that in all periods \( t \), the demand must be taken for each node \( i \)

\[
\left( \sum_k \left( S_{ik} \cdot \sum_{op} f_{k,op} \right) \right) + D_i = 0
\]

For all the branches \( k \) in any option \( op \) Eq.(4) is necessary:
The radial configuration of the network and proper treatment of all demand points are guaranteed by requiring that in each period \( t \), Eq.(5) must be satisfied:

\[
R_{op} \cdot f_k \cdot s_k \cdot V_{k} \leq M \cdot \left(1 - y_{k, op} \right).
\]

For every node \( i \) with nonzero demand Eq. (6) values:

\[
\sum_{k} \sum_{op} y_{k, op, t} \geq ND_i.
\]

Each branch can be used only in one direction and with a single conductor option. Operational limits of the equipment should be respected for substations and for conductors. The variables associated with investments should be related to the need of the use of the facility and to be subject to corresponding conditions which prevent repeated or obsolete investments. Then \( x_{k, op, t} = 0 \) is considered for all branches \( k \) of the network that are installed on the option \( op \) in the initial network in any period \( t \). If in the first period of planning there is a need to use the branch \( k \), the investment must be done as soon as possible and \( x_{k, op, t} = y_{k, op, t} \). In the next periods the investment in the branch \( k \) will be necessary if it has not been done earlier for this same option or better options. Similar restrictions are imposed on investment variables associated to the substations.

Some logical constraints are necessary for not making the choice of more than one option \( op \) for the same facility in a given period \( t \). To ensure service quality, the tensions in all network nodes must be kept within limits previously established.

The quadratic mixed integer multicriteria model thus obtained for the optimization of multistage expansion planning of distribution systems of electric power can be solved by applying mathematical programming techniques. The determination of non-dominated solutions can be made through the method of weights where each of the mono-objective problems obtained can be solved using techniques like branch-and-bound.

2.1 Results with an exact execution

Validation of mathematical model presented was made by applying in a network presented in [8], consisting of 18 nodes (2 substations and 16 demand nodes) and 24 branches operating in 13800 V. Three stages were considered. The results were obtained using the LINGO 9.0 solver executed on a computer with AMD Turion (tm) 64 Mobile, Technology MK-38, 2.20 GHz, 1.93GB of RAM and Windows XP Professional operating system, Version 2002, Service Pack 3.

The method of weights for multiobjective problem gives points of the Pareto frontier of the problem through different choices of positive parameters in the weighting of the objective function. The solutions obtained have characteristics very consistent with expected according to the nature of each objective investigation.

Figure 1 shows some points of the efficient frontier of the multiobjective model formulated in this paper applied to the primary network for distribution of electricity applied to the fictitious network presented in [8].

3. Particle Swarm Optimization model

Although the exact model has provided very interesting solutions, it is evident that its application is restricted to very small networks. Aiming to facilitate the planning for the expansion of distribution centers for great consumers we search for a heuristic methodology, resulting in solutions that give a good approximation of the
Pareto frontier for the problem in reasonable computational time. We opted for the use of metaheuristic PSO due to its simplicity and efficiency presented in the literature, like in [3].

3.1 Particle Swarm Optimization – PSO methodology

Particle Swarm Optimization – PSO is a relatively recent heuristic inspired by the choreography of a bird flock or fish school searching for food, combining principles of social psychology and evolutionary computation. PSO was originally conceived and developed by Eberhart and Kennedy in 1995. In the PSO algorithm each particle represents a possible solution and is associated with two vectors, the position \( x \) and velocity \( v \). In a search space \( n \)-dimensional vectors \( x_i = (x_{i1}, x_{i2}, \ldots, x_{in}) \) and \( v_i = (v_{i1}, v_{i2}, \ldots, v_{in}) \) are both associated with the particle. A swarm consists of a number of particles that "fly" through space to explore viable solutions to optimal solutions. Each particle updates its position and its speed based on his best performance (a position that gives you the best value of objective function) and also in the best performance of the swarm. The update happens according to the Eq. (7) and Eq. (8) and the notation presented in Table 2.

\[
v_{i}^{k+1} = \omega \cdot v_{i}^{k} + c_1 \cdot r_1 \cdot \left( pbest_{i}^{k} - x_{i}^{k} \right) + c_2 \cdot r_2 \cdot \left( gbest^{k} - x_{i}^{k} \right)
\]

\[
x_{i}^{k+1} = x_{i}^{k} + v_{i}^{k+1}
\]

Table 2: Notation used in Equations (7) and (8).

<table>
<thead>
<tr>
<th>Indices and variables:</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k )</td>
<td>iteration index</td>
</tr>
<tr>
<td>( c_1, c_2 )</td>
<td>positive constants</td>
</tr>
<tr>
<td>( r_1, r_2 )</td>
<td>randomly generated numbers in ([0,1])</td>
</tr>
<tr>
<td>( \omega )</td>
<td>inertia weight</td>
</tr>
<tr>
<td>( pbest_{i}^{k} = (x_{i1}^{pbest}, x_{i2}^{pbest}, \ldots, x_{in}^{pbest}) )</td>
<td>best position of the particle ( i ) until ( k ) iteration.</td>
</tr>
<tr>
<td>( gbest^{k} = (x_1^{gbest}, x_2^{gbest}, \ldots, x_n^{gbest}) )</td>
<td>best position among all particles of the swarm until the iteration ( k ).</td>
</tr>
</tbody>
</table>

Eq. (7) determines the direction in which the particle \( i \) will move and has three distinct components:

- The first component models the tendency of the particle to continue in the same direction as it was, it refers to the inertia of the particle. This component was introduced in PSO by Shi and Eberhart in 1998.
- The second component refers to the memory of the particle, the interest in following the direction of their individual best position.
- The third component refers to the social cooperation of the swarm, the tendency of the particle to follow the direction of the best position found by all particles of the swarm.

The position of the \( i \)-th particle is then updated by Eq. (8).

The PSO was originally designed to solve optimization problems whose search space is continuous, after Eberhart and Kennedy in 1997 developed a modified version, called Binary Particle Swarm Optimization (BPSO) that can be used to solve combinatorial problems. In the paper [3] Coello extends the heuristic PSO to multiobjective optimization problems. The proposal is based on the idea of an external memory to store Pareto optimal solution. The best position of the particle (\( pbest \)) and of the swarm (\( gbest \)) is chosen using the concept of Pareto dominance.

Del Valle in her paper [4] presents an extensive list of publications where PSO has been applied to solve optimization problems in power systems and suggests that this technique can be extended to applications in power distribution systems layout, such as deciding optimal locations of substations and size or topology of the cable system.

3.2 Application of Particle Swarm Optimization

The application of metaheuristic for solving the problem requires that we code the solution, which will take place updates in search of better solutions for the objectives established. As any other metaheuristic, PSO is characterized as a search strategy in the space of solutions, with strong random components. The necessity for radial configuration for the network is presented as a rather restrictive condition for the search space. When applying the strategy of updating the solutions in a known solution, no one has, in principle, ensuring that the solution is feasible, which happens frequently is the radiality is disregarded. This suggests that the application of metaheuristic code directly into the solution is not good enough since it creates the need for a process to become
feasible these solutions and this process can become computationally expensive and destroy the randomness of the methodology. In an attempt to escape this pattern updates which found to be infeasible, a network with \( k \) possible branches, we can associate a vector of \( k \) random numbers and a feasible solution to the problem can be found, with a number of failures significantly lower. Furthermore, small changes in these random numbers cause small changes in the associated solution, which brings us to the idea of neighborhood used in many heuristics. From this association we can upgrade the solution proposed by the heuristic chosen in the vector of random numbers. The reduction of infeasibility that arises during the resolution of the problem is an important attraction of this methodology that is being proposed. It is considered interesting to note also that the problem of planning has binary solution of the independent variables to represent the use of branches and substations. However, applying the procedure proposed here allows the use of continuous versions of the metaheuristic. In the particular case of PSO, the continuous version uses more clearly the information of the previous solution when compared to the binary version of PSO, a fact that makes the search more efficient.

Using this methodology we chose to apply the version of multiobjective PSO by [3]. At the time of initial generation of the swarm, each particle is determined using the following sequence of procedures:

a. Generation of a vector of random numbers, \( nurand(p) \).

b. Determination of network topology associated with this vector.

c. Determination of electrical network configuration using the topology defined in the previous item.

d. Calculation of the dependent variables (the flow in each branch, each node voltage, variable investment) and the objective functions for the solution \( fit(p) \).

When the particles are update, the rule is applied in \( nurand(p) \) and after the update is determined the new solution associated with this \( nurand_{atualizado} \) using the same sequence of procedures listed above.

The procedure for the association of a feasible solution of the problem to a given vector of random numbers is performed in two stages. The first step refers to the combination of a radial topology of the vector of random numbers while the second comprises the determination of network configuration already glimpsing the objectives proposed for the problem. For clearing up, the network topology information choose the branches that will used in each period while planning choose the network configuration including information about the gauge of the conductor to be used in every branch and capacity of substations in each period.

The association process of a radial topology to an array of random numbers involves a linear optimization problem of very simple resolution. This step of the methodology is denominated \( \text{GERARVORES} \) and the notation used is in Table 3.

| Binary decision variables: | Meaning | | | |
|---------------------------|---------|---------|---------|
| \( arv_k,t \)             | binary variable associated with the decision to use the branch \( k \) in the period \( t \) | | | |
| \( zz_j,t \)              | binary variable associated with use of the substation \( j \) in the period \( t \) | | | |
| Real variables:           | Meaning | Unity |
| \( ff_k,t \)              | flow through the branch \( k \) in the period \( t \) | A |
| Data:                     | Meaning | Unity |
| \( nurand \)              | vector of \( k \) random numbers | | |
| \( f\text{MAX} \)         | maximum flow on a branch | A |
| \( cap\text{MAX} \)       | maximum capacity of any substation | A |
| \( D_i,t \)               | demand forecast at the node \( i \) for the period \( t \) | A |
| \( ND \)                  | number of nodes with nonzero demand in the period \( t \) | | |
| \( K \)                   | number of possible branches of the network | | |
| \( SS \)                  | incidence matrix of the network, with duplicated branches | | |
| \( SS_{hk} = -1 \)        | means that the branch \( k \) ends at the node \( h \) | | |
| \( SS_{hk} = 1 \)        | means that the branch \( k \) originates at the node \( h \) | | |
| \( l_k \)                 | length of branch \( k \) | km |

Given the vector \( nurand \) of \( k \) random numbers, the problem can be formulated as appeared in Eq. (9) to (20) as follows:

\[
\text{Minimize} \quad \sum_t \sum_k nurand_k \cdot arv_{k,t}.
\] (9)
Subject to, in all periods:

Radial configuration  \[ \sum_k |arv_{k,t}| \geq ND_t. \]  

Each node with nonzero demand must be served and the service should be just by one branch  \[ \sum_{k \in S} |arv_{k,t}| = 1. \]  

Only one direction for each arc  \[ arv_{k,t} + arv_{k,t+k} \leq 1. \]  

The flow can only leave any substation, then for every branch that reaches any substation we must have  \[ arv_{k,t} = 0. \]  

The branches can only leave nodes that are substations or that have a branch coming; then for every node of demand,  \[ \sum_{k \in S} |arv_{k,t}| \leq M \cdot \sum_{k \in S} |arv_{k,t}|. \]  

Kirchhoff's Law assigning a flow through a branch, independent of the gauge of the conductor in each node  \[ D_{h,t} + \sum_k |SS_k \cdot ff_{k,t}| = 0. \]  

Capacity limit for conductors for the largest possible value for all branches  \[ ff_{k,t} \leq fMAX \cdot |arv_{k,t}|. \]  

Capacity of substations for the largest possible for all substations  \[ \sum_k |ff_{k,t}| \leq capMAX \cdot |zz_{j,t}|. \]  

Every substation installed must be connected to some demand node  \[ \sum_{k \in S} |arv_{k,t}| \geq |zz_{j,t}|. \]  

Do not ignore substations already installed  \[ |zz_{j,t}| \geq SBT_{j,t}. \]  

You can make the choice not to allow changes in branches that were initially installed  \[ arv_{k,t} + arv_{k,t+k} = 1. \]  

The second part of the association of a feasible solution to the vector of random numbers corresponds to the choice of size for the conductors in each branch used in the topology adopted in the first part. This choice is directly related to the objectives initially imposed to the problem of planning. The size conductor’s strongly influences the cost of investment in each section and the amount of energy lost during the distribution process. When using the objective function related to electrical losses falls on the problem of nonlinearity of the exact model, which greatly increases the time resolution of the stage. For this reason we chose to include a linear term in the objective function whose minimization results in loss reduction. This step also results in a linear minimization problem and is denominated SOLFAC. The notation of this formulation is in Table 4.

<table>
<thead>
<tr>
<th>Binary decision variables:</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y_{k,op,t} )</td>
<td>binary variable associated with the decision to use the branch ( k ), at the option ( op ) in the period ( t )</td>
</tr>
<tr>
<td>( z_{j,cap,t} )</td>
<td>binary variable associated with use of the substation ( j ), with the capacity ( cap ), in the period ( t )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Real variables:</th>
<th>Meaning</th>
<th>Unity</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_{h,t} )</td>
<td>voltage at node ( h ) in period ( t )</td>
<td>( V )</td>
</tr>
<tr>
<td>( f_{k,op,t} )</td>
<td>current flowing through the branch ( k ), installed in the option ( op ), in the period ( t )</td>
<td>( A )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Data:</th>
<th>Meaning</th>
<th>Unity</th>
</tr>
</thead>
<tbody>
<tr>
<td>( O_{k,op} )</td>
<td>cost of operation/maintenance of the branch ( k ) on option ( op )</td>
<td>( S )</td>
</tr>
<tr>
<td>( OS_{j,cap} )</td>
<td>cost of operation/maintenance of the substation ( j ) with installed capacity ( cap )</td>
<td>( S )</td>
</tr>
</tbody>
</table>

Table 4: Notation used in SOLFAC.
Given the vector \( arv \), obtained in \( GERARVORE \), the problem can be formulated as appeared on Eq. (21) to (28) as follows:

Minimize

\[
\sum_{k} \sum_{op} \left[ \sum_{t} A_{1} \cdot (CIR_{k,op} \cdot l_{k} + O_{k,op}) + A_{2} \cdot R_{op} \right] \cdot y_{k,op,t} + \sum_{j} \sum_{cap} A_{1} \cdot (CIS_{j,cap} + OS_{j,cap}) \cdot z_{j,cap,t} \]
\]

Subject to:

Kirchoff’s flow laws for all periods \( t \) and all demand nodes \( i \)

\[
\sum_{k} \left( S_{k,i} \cdot f_{k,op,t} \right) + D_{i,t} = 0
\]

(22)

Kirchoff’s voltages law, for all periods \( t \), for all branches \( k \) and for all size \( op \)

\[
R_{op} \cdot l_{k} \cdot f_{k,op,t} - \sum_{k} S_{k,k} \cdot V_{k} \leq M \cdot \left( 1 - y_{k,op,t} \right)
\]

(23)

For all periods \( t \), for all branches \( k \) and for all size \( op \) the flow must respect the capacity of the conductor

\[
f_{k,op,t} \leq y_{k,op,t} \cdot f_{MAX_{op}}
\]

(24)

For all periods \( t \) the installed capacity of the substation must be superior of the total demand

\[
\sum_{j} \sum_{cap} cap \cdot z_{j,cap,t} \geq \sum_{i} D_{i,t}
\]

(25)

For all periods \( t \), each substation can be installed with only one capacity

\[
\sum_{cap} z_{j,cap,t} \leq 1
\]

(26)

For all periods \( t \), the capacity bound of each substation \( j \) must be considered

\[
\sum_{k} \sum_{op} f_{k,op,t} \leq \sum_{cap} cap \cdot z_{j,cap,t}
\]

(27)

The network topology obtained in \( GERARVORE \) must be used in all periods \( t \). Then, for all branches \( k \)

\[
\sum_{op} y_{k,op,t} = arv_{k,t}
\]

(28)

The application of Particle Swarm Optimization metaheuristic combine with SOLFAC in the problem of electric planning results in a promise methodology for problems of this kind.

3.3 Results obtained by applying the proposed methodology

Aiming to evaluate the efficiency of the methodology that is proposed in this work was held the first tests with the same network used in the fictitious three-phase validation of the exact model presented in paragraph 2. The results show the vocation to the success of the proposed methodology. Admittedly, many adjustments must still be made. The results were obtained using the solver LINGO 9.0 and MatLab R2009a running on a computer with an Intel Core 2Duo, 2 GHz, 3GB of RAM and Windows Vista operating system.

The use of 20 particles and 150 iterations only showed the tendency of non-dominated solutions found during the search process, approaching the efficient frontier determined by the exact model.
The solutions exhibit characteristics consistent with that expected according to the defined objectives. Changes in the original network and the choice of the size were made with the highest weight to the cost function. In figures 2 and 3, the red little crosses represent the points of the swarm in the iteration and blue dots represent the non-dominated solutions accumulated during the iterations and green balls represent the efficient frontier determined by the exact model.

**Figure 2:** Graph comparing the exact efficient frontier (cost x losses) with an initial particle swarm randomly generated.

**Figure 3:** Evolution of the swarm after 150 iterations.

It is believed that a better fit in the parameters of weighting of goals when the choice of size for conductors and also the weighting factors of cognitive influences, the social upgrading of the solutions may result in borders closer to the exact border. The figure 4 represents the network topology in each stage of the horizon planning. The conductor size of each branch is identified by the thinness of the line. The total cost and the technical losses of this solution are respectively $3127.20 \text{ e 865313.0 W.}$

**Figure 4:** Topology of one solution in efficient frontier (cost x losses) obtained using PSO.
References