Critical study of design parameterization in topology optimization; The influence of design parameterization on local minima

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Abstract

In the past decade, SIMP-based topology optimization methods have become increasingly popular. They offer the designer the possibility to perform design optimization in an early stage of the design process for a wide range of applications. The specific design parameterization used in SIMP is the result of many attempts to eliminate numerical artifacts in the results of topology optimization problems. However, any choice of design parameterization also influences the shape of the response functions. For a gradient-based optimization method it is important to obtain a smooth and, if possible, convex response. This research aims to point out relevant consequences of using the SIMP design parameterization in combination with density filters on the response functions.

In this contribution we show the effects of the power of the penalization and the size of the density filter on the response functions. The introduction of penalization has two effects. Firstly, increasing penalization introduces an increasingly strong immobility of the boundary of the structure. Secondly, the introduction of a density filter is not only useful to avoid checkerboard patterns, control minimum member size and achieve mesh-independence, but it also reduces the immobilizing effect that penalization has on the mobility of the boundary of the structure. This effect and its dependence on the power of the penalization and the size of the density filter are illustrated using several elementary numerical examples involving compliance minimization. The results are expected to be relevant for any type of structural topology optimization.

Keywords: Topology optimization, design parameterization, penalization, density filter, local minima

1 Introduction

These days, topology optimization is a very useful tool in the conceptual design phase. It allows the designer to perform a very flexible design optimization in an early stage of the design process, creating a design concept in terms of a general material distribution (solid vs. void). However, throughout the history of topology optimization one of the biggest challenges has been to come up with a design parameterization that leads to physically optimal designs [1–3]. In this case physically optimal means the optimal design that is desired by the engineer.

Topology optimization problems can contain an extremely large number of design variables. Therefore, it is difficult to explore the entire design space. In order to enable to use of gradient-based solvers the integer-based (black-and-white) topology optimization problem can be relaxed to a formulation based on artificial element densities [4]. Density-based topology optimization methods use the material fraction per finite element, called element densities, as design parameterization. These element densities are then used to scale the stiffness of the corresponding finite elements.

The results of compliance minimization problems using this design parameterization include large areas with intermediate densities. These intermediate densities can be interpreted as material with a porous microstructure [3, 5, 6]. However, these optimal solution can be impossible to fabricate because of this microstructure. Therefore, even though this is numerically optimal, from a designer's point of view this is an unwanted effect. Therefore, new types of design parameterization were formulated that somehow penalize intermediate values of the element densities [5, 7–10]. The most popular design parameterization, Solid Isotropic Material with Penalization (SIMP), replaces the linear relation between the element densities and the stiffness of the finite element with an exponential one [5, 7]. The cost of having an intermediate stiffness in a finite element is now relatively high compared to full material elements.

Even after introducing penalization, the resulting optimal designs contain yet another type of numerical artifact related to the finite element discretization; checkerboard patterns [1, 2, 11]. These patterns are not a physically sound representation of a design, but from a numerical point of view optimal. In order to eliminate these artifacts from the results of the topology optimization a large number of adjustments have been proposed [1, 3]. For instance, sensitivity (mesh-independence) filtering [12] prevents the topology optimization from arriving in a checkboard configuration. Other methods include perimeter-control [13, 14], slope-constrained density fields [15], an adjusted mixed formulation [11], level-set-based topology optimization [16, 17] and others [2, 18, 19]. One of the most popular solutions to avoid these checkerboard patterns is the usage of density filters [20, 21]. When the stiffness in an element is determined based on a weighted average of the element densities in its neighborhood, checkerboard patterns can no longer emerge.

In parallel, gradient-based methods have been developed especially for topology optimization. Due to the large number of design variables it is very expensive to calculate and store second order information of the response functions. However, it is possible to approximate the second order information of the response functions using available knowledge about the physics involved. These algorithms include the Method of Moving Asymptotes (MMA) [22], COnvex LINearization method (CONLIN) [23] and sequential quadratic approximations [24]. Using these fast optimization algorithms and the successful combination of penalization and density filters, a wide range of topology optimization problems can be solved.

Summarizing, the development of topology optimization methods has been focused mainly on two subjects:

- Solve efficiently with a gradient-based algorithm
- Find a design parameterization that enables us to find the physically optimal solution

Therefore, it is interesting to see what consequences the design parameterization has in relation to a gradient-based solution. We do not want to introduce artificial local optima into the problem. This paper aims to illustrate the effect that the penalization and filtering schemes have on the shape of the response functions.

In Section 2 the formulation which is used in this paper is presented. The different effects of the penalization and the density filters are compared in Section 3. Finally this paper ends with the conclusions and recommendations in Section 4.

2 Topology optimization framework

The most common design parameterization used to formulate a topology optimization today is SIMP [5, 7]. In this section we provide the framework that we use in this paper for the numerical examples. To parametrize the structural domain of a possible design, element densities ρ_e are formulated indicating whether a finite element is (partly) part of the structural domain. These element densities are used to scale the stiffness tensor in each element \mathbf{C}_e simulating material and void,

$$\mathbf{C}_e = \rho_e^p \mathbf{C},\tag{1}$$

where **C** is the original stiffness tensor of full material and the exponent p is the penalization used to make intermediate densities unfavorable in terms of stiffness. This formulation is a slightly altered version of the conventional exponential relation [3, 6, 25]. The bounds of the 'effective' element density ρ_e^p are chosen such that the structural problem does not become singular,

$$\varepsilon \le \rho_e^p \le 1 \tag{2}$$

where $\varepsilon = 10^{-6}$ is the lower bound. A common penalization exponent for compliance minimization is p = 3 (for compliant mechanism design this is often p = 4) [3, 6, 25].

This parameterization is augmented with a filtering strategy to avoid numerical artifacts in the optimal solution. Instead of the element density ρ_e , we now use the filtered element density $\tilde{\rho}_e$ in Eq. (1). This filtered element density $\tilde{\rho}_e$ is the weighted mean of the element densities in the neighborhood N_e of the element [19, 20]. This neighborhood is defined as,

$$N_e := \{i \mid \|\mathbf{x}_i - \mathbf{x}_e\| \le R\},\tag{3}$$

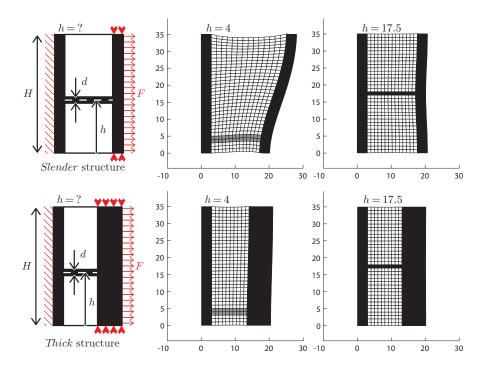


Figure 1: The loading conditions of the *slender* and the *thick* H-shaped structure with d = 1 (one element). The deformations of the *slender* structure are relatively larger than those of the *thick* structure.

where R is the filter radius. The definition of a filtered element density $\tilde{\rho}_e$ is given by,

$$\tilde{\rho}_e = \frac{\sum_{i \in N_e} w(\mathbf{x}_i, \mathbf{x}_e) \rho_i}{\sum_{i \in N_e} w(\mathbf{x}_i, \mathbf{x}_e)},\tag{4}$$

where $w(\mathbf{x}_i, \mathbf{x}_e)$ is a weighting function. In this paper a *linear* weighting function is used which is defined as,

$$w(\mathbf{x}_i, \mathbf{x}_e) = R - \|\mathbf{x}_i - \mathbf{x}_e\|. \tag{5}$$

This design parameterization should make sure that the optimal distribution of material is also physically optimal; the optimal design that is desired by the engineer. Using this framework we can investigate the shape and convexity of the response function used in compliance minimization problems.

3 Comparison

Both penalization and as well as filtering have an effect on the shape of the response functions. In this paper we will focus on the effect on compliance minimization, but it is expected that the effects on any type of structural topology optimization problem will be similar.

Two similar illustrative structures, see Figure 1, are used to demonstrate the type of effect that penalization and filtering have on an objective function; a relatively *slender* and *thick* structure. Both cases deal with an H-shaped structure which is forced to the right by a distributed load. The left side of the H-shape is clamped and the right side can only slide in horizontal direction. We are looking for the position of the horizontal member h which gives the minimum compliance of these structures. The height of the structure and the thickness of the horizontal member are denoted by H and d, respectively.

The 2D finite element computation is performed using square quadrilaterals. The 20x45 design domain is meshed with 20x45 finite elements. The structure is modeled using plane-stress linear elasticity with a Young's modulus of E=100 and Poissons's ratio of $\nu=0.3$.

We are interested in the shape of the objective as a function of the location of the horizontal member in vertical direction f(h) when the amount of material is kept constant (d = constant). Physically, we can

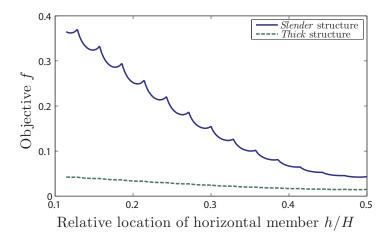


Figure 2: The *slender* and *thick* structures with d = 1 (one element) and no penalization, p = 1, or filtering, R = 0, have a relatively steep and flat objective, respectively.

expect both structures to have a smooth response with respect to the location of the horizontal member. The (bending and tensional) stiffness of this member does not depend on the location of the structural member. The physical optimum can be found when the horizontal member is situated exactly in the middle of the structure.

However, the *computational* stiffness of the rod does depend on the position of the horizontal member relative to the discretized mesh and the design interpolation. Therefore, any local minima that may be found in the response of the discretized system is therefore *artificial*; either due to the discretization and/or the design parameterization.

These examples will show what happens to the compliance when a small member of a structure is moving through the mesh under a volume constraint. Ideally we would like to see a smooth response, that will enable the gradient-based solver to find the physical optimum.

3.1 Without penalization or filtering

First we observe the behavior of the structures without penalization or filtering. The objectives corresponding to these two structures with d=1 (one element) have different shapes, see Figure 2. The structure is symmetric in the vertical direction. Therefore only half of the graph is shown, $h/H \leq .5$.

The deformation of the *slender* structure depends much on the position of the horizontal member. When the horizontal member is placed eccentric, bending will induce large displacements and a high compliance, see Figure 1. Therefore, the compliance of the *slender* structure depends *strongly* on the position of the horizontal member h. It also depends on the bending stiffness of the horizontal member, especially when it is placed eccentrically. A position of the member with respect to the mesh which is such that the material is divided over multiple elements is favorable for its bending stiffness. Because of this the objective shows local minima for small values of h/H in Figure 2.

The thick structure has a lot of intrinsic bending stiffness due to the thick right vertical member. This part of the structure offers resistance against the bending deformation and the resulting displacements will be smaller. Therefore, the compliance of the thick structure depends weakly on the position of the horizontal member h, see Figure 1. Furthermore, the tensional stiffness of the rod is now more important than the bending stiffness. Therefore, a position of the member with respect to the mesh which divides the material over multiple elements is no longer very favorable for the compliance. Therefore, the objective does not show clear local minima on the left side of Figure 2.

Because of the wiggles, we see that even without penalization the shape of the objective can depend on the discretization and the design interpolation. In this case the compliance minimization favors intermediate densities because of its relatively high bending stiffness, as observed in standard benchmark problems. From Figure 2 it is clear that a topology optimization may get stuck in a computational local optimum far from the physical global optimum.

However, the current example treats a rod moving through a finite element mesh. In a density-based

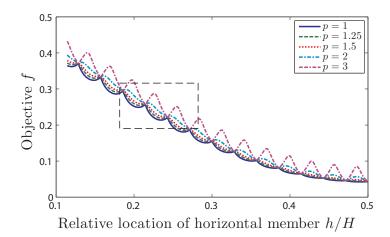


Figure 3: The compliance of the *slender* structure as a function of the position of the horizontal member f(h) with different penalization exponents $p = \{1, 1.25, 1.5, 2, 3\}$. The thickness of the horizontal member d = 1 is exactly the height of one finite element.

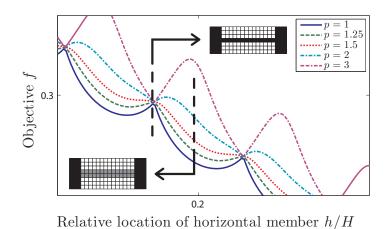


Figure 4: Close-up of part of Figure 3. Penalization changes the objective for the intermediate densities.

topology optimization there is more freedom for the design to change in a diffusion-like manner. Still it is important to realize that a small member is unable to move through the mesh as a whole. To eliminate the preference for intermediate densities, the usual approach is to penalize these intermediate densities using an exponential relation between the element density and the element stiffness.

3.2 Effect of penalization

When we include penalization, the stiffness of the intermediate densities becomes relatively low. In Figure 3 the compliance of the *slender* structure with d=1 (one element) is plotted as a function of the position of the horizontal member f(h) for different penalization exponents $p=\{1,\ 1.25,\ 1.5,\ 2,\ 3\}$. In Figure 4 a close-up is shown.

For all positions where the horizontal member is discretized with full densities, the performance f(h) does not depend on the penalization exponent p, since $1^p = 1$. In between, the compliance f(h) increases with increasing penalization exponent p. Without penalization p = 1, intermediate densities were favorable for a low compliance and with the usual penalization p = 3 [REF] the intermediate densities are unfavorable. In both cases the objective contains very apparent local minima.

Somewhere between these two extremes 1 the objective becomes more or less smooth.

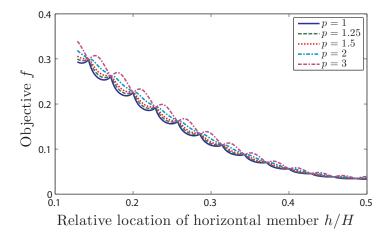


Figure 5: The compliance of the *slender* structure as a function of the position of the horizontal member f(h) with different penalization exponents $p = \{1, 1.25, 1.5, 2, 3\}$. The thickness of the horizontal member d = 2 is now exactly the height of two finite elements.

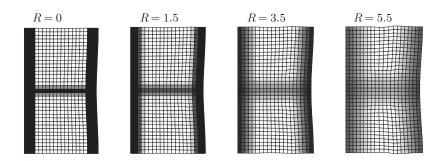


Figure 6: The densities after filtering, $\tilde{\rho}_e$, of the *slender* structure with different filter radii $R = \{0, 1.5, 2.5, 5.5\}$. The thickness of the horizontal member d = 2 is exactly the height of two finite elements.

However, for none of the choices of the penalization exponent p the objective becomes convex. Near the physical optimal solution the objective becomes more flat and the wiggles always result in other local minima.

Another observation is that this effect on the objective becomes more apparent for smaller member sizes d. When we take a look at the same graph for a horizontal member with twice the thickness d=2 (two elements), the wiggles have become visibly smaller, especially for a high penalization p=3, see Figure 5. Of course, the magnitude of the wiggles depend on the type of structural problem. It also depends on the number of densities that are changing from full material to intermediate densities as the member moves through the mesh.

In a topology optimization, a configuration with a small member would be stuck in a local optimum. Since the intermediate densities are penalized, there is no way for material to move more freely. In general, the application of only penalization leads to new numerical artifacts, such as checkerboard patterns. The addition of filtering to the design interpolation solves this problem, but also has an effect on the shape of the response functions.

3.3 Effect of filtering

The application of a filtering scheme ensures that there will always be a band of intermediate densities, independent of the member size. Increasing the filter radii R will increasingly blur the structure and increase the number of intermediate densities in the structural analysis, see Figure 6.

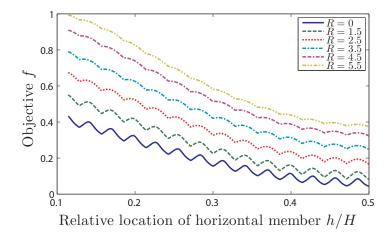


Figure 7: The compliance of the *slender* structure as a function of the position of the horizontal member f(h) with penalization exponent p=3 with different filter radii $R=\{0,\ 1.5,\ 2.5,\ 3.5,\ 4.5,\ 5.5\}$. The thickness of the horizontal member d=1 is exactly the height of a finite element.

Of course, this affects the shape of the objective function. In Figure 7 the compliance of the slender structure using a penalization exponent p=3 is plotted as a function of the position of the horizontal member f(h) for different radii of a linear filtering scheme $R = \{0, 1.5, 2.5, 3.5, 4.5, 5.5\}$. Not only does the filter solve the problem of numerical artifacts, but it also smooths the objective function f(h). Of course the compliance is generally higher when we increase the filter size R; more elements will have an intermediate density which is penalized in terms of stiffness because of the penalization exponent p=3.

As stated before, the wiggles generally depend on the number of changing intermediate densities and the number of material elements. With a large filter radius R, the amount of intermediate elements now stays more or less constant as the member moves through the mesh. Therefore, the number of elements with a relatively bad or good performance does not change as much as before. The resulting smooth behavior of the response function will have a good effect on the convergence of a gradient-based solver. The numerical optimum is now useful for a designer because it is close to the physical optimum.

When we look at the compliance of the *slender* structure with a horizontal member size d=2 of the height of two finite elements, the effect of the filtering is even more apparent, see Figure 8. The bottom line indicating the compliance without any filtering R=0 has some relatively large wiggles. But even the smallest filter size R=1.5 already takes care of most of these wiggles. A gradient-based algorithm would have no trouble finding the desired physical optimal solution to the topology optimization problem.

It is also interesting to take a look at the shape of the objective function when we apply filtering to the non-penalized formulation p=1. In Figure 9 the compliance f(h) is displayed of the slender structure with a horizontal member size d=1 (one element) for different filter radii $R=\{0,\ 1.5,\ 2.5,\ 3.5\}$. The line on the top of the graph indicating the compliance without filtering still has many wiggles. Also for this case the application of a linear filter has a smoothing effect on the objective function. A difference though, is that in this case the filtering actually improves the performance, since the increased number of intermediate elements perform relatively well. However, this design interpolation is not very useful in a topology optimization, because it would result in large areas of intermediate densities. Intermediate densities are still favored with respect to their relatively high bending stiffness.

In general the combined effect of penalization rand filtering reduces the number of local optima that may be created because of the design interpolation and discretization. These procedures have been proposed to obtain computational optimal designs that are as close as possible to the desired physical optimal solution. However, not only the end result of the topology optimization is influenced, but also the smoothness of the objective function which leads to this end result.

Finally, it is stressed that the number of local minima introduced by the design interpolation depends on the relative size of the wiggles with respect to the objective. In Figure 10, the compliance of the thick structure with a horizontal member size d=1 with a penalization exponent of p=3 is displayed for different filter radii $R=\{0,\ 1.5,\ 2.5,\ 3.5\}$. There is not much bending because of the high bending stiffness of the thick vertical right member. Therefore the compliance is less sensitive to the exact location

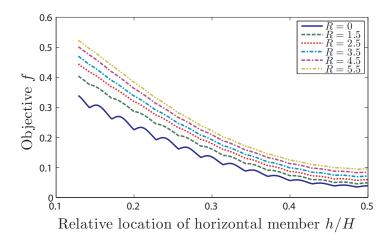


Figure 8: The compliance of the *slender* structure as a function of the position of the horizontal member f(h) with penalization exponent p=3 with different filter radii $R=\{0,\ 1.5,\ 2.5,\ 3.5,\ 4.5,\ 5.5\}$. The thickness of the horizontal member d=2 is exactly the height of two finite elements.

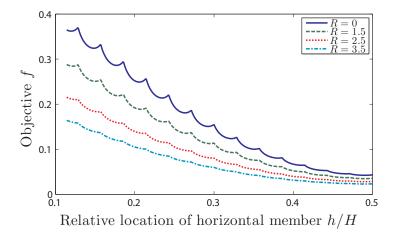


Figure 9: The compliance of the *slender* structure as a function of the position of the horizontal member f(h) without penalization p=1 with different filter radii $R=\{0,\ 1.5,\ 2.5,\ 3.5,\ 4.5,\ 5.5\}$. The thickness of the horizontal member d=1 is exactly the height of a finite element.

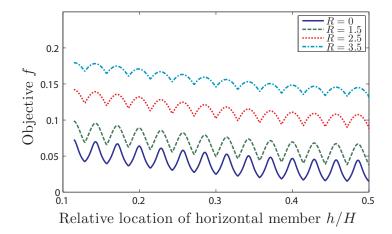


Figure 10: The compliance of the *thick* structure as a function of the position of the horizontal member f(h) with penalization exponent p=3 with different filter radii $R=\{0,\ 1.5,\ 2.5,\ 3.5\}$. The thickness of the horizontal member d=1 is exactly the height of a finite element.

of the horizontal member h. However, it is sensitive to the effective stiffness due to the relative position of the member in the mesh. The effective stiffness is reduced when a large part of the material of the member is discretized with intermediate densities. Still the wiggles are reduced when the filter radius R is increased but relative to the 'physical' objective these oscillations remain large. In this case a gradient based algorithm may get stuck in an artificial local minimum irrespective of the filter radius R.

4 Conclusions & recommendations

It is important that a structural topology optimization yields computational optima that are close to the desired physical optima. Not only does penalization and density filtering help finding the desired optima, but it also smooths any wiggles that may appear in the response functions due to the design parameterization on a finite element mesh. Especially filtering techniques will have a positive effect on the convergence of a gradient-based topology optimization method.

In order to eliminate numerical artifacts, the design parameterization is often altered to include the penalization of intermediate densities. Without penalization, intermediate densities are favored, and with penalization whole densities are favored in compliance minimization. This introduces mesh dependent wiggles and even additional local minima in the response functions of the topology optimization. The inclusion of filters in the design interpolation has a smoothing effect on the response functions. Therefore, the popularity of the combination of penalization and filtering is not surprising.

However, in general the design interpolation may always have an influence on the shape of the response functions. Especially in flat regions of the response functions the design interpolation may introduce artificial local minima. This effect is worst when some members of the design are small.

However, in an actual topology optimization the design may have more freedom to change than in the examples treated in this paper. However, especially in the final stage of a topology optimization this effect will be apparent. It is important to be aware of the influences that the formulation of the design parameterization may have on the response functions.

Furthermore, another way of eliminating the numerical artifacts is sensitivity filtering. This results in an inconsistent topology optimization problem. However, both the structural analysis and the sensitivities come from different numerical versions of the same physical problem. The success of this method may lie in fact that the inconsistencies are able to jump these artificial local minima. However, this will also result in problems finding a physical optimal solution to the optimization problem.

Acknowledgments

The authors would like to acknowledge the funding of the Dutch MicroNed programme which made this research possible.

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