Modeling and Multi-Objective Optimization of Cyclone Vortex Finder using CFD and Neural Networks

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Abstract

Vortex finder is a key part of cyclone separator. In the present study, multi-objective optimization of vortex finder is performed at three steps. At the first step, collection efficiency (η) and the pressure drop (∆p) in a set of cyclones with different vortex finder shape are numerically investigated using CFD techniques. Two meta-models based on the evolved group method of data handling (GMDH) type neural networks are obtained, at the second step, for modeling of η and ∆p with respect to geometrical design variables. Finally, using obtained polynomial neural networks, multi-objective genetic algorithms are used for Pareto based optimization of vortex finder considering two conflicting objectives, η and ∆p. It is shown that some interesting and important relationships as useful optimal design principles involved in the performance of cyclones can be discovered by Pareto based multi-objective optimization of the obtained polynomial meta-models. Such important optimal principles would not have been obtained without the use of both GMDH type neural network modeling and the Pareto optimization approach.

Keywords: Gas-Solid; Vortex Finder; Multi-Objective Optimization; CFD; GMDH.

1. Introduction

Cyclones are widely used in Filtration and Separation industries as a result of their low setup and maintenance cost. In conventional cylindrical cyclone devices, there are two outlets, both on the axis of symmetry. The underflow outlet is situated at the apex of the cone at the base of the cyclone, and the over flow outlet is an inner tube (or so-called vortex finder) that descends from the top, as shown in Figure 1.

![Vortex Finder](image)

Figure 1: Situation of the vortex finder in cyclone body.
Very little information is available on the effects of the vortex finder size and shape, if other cyclone dimensions are fixed. The vortex finder size is an especially important dimension, which significantly affects the cyclone performance as its size plays a critical role in defining the flow field inside the cyclone, including the pattern of the outer and inner spiral flows. Optimization of cyclone vortex finder is, indeed, a multi-objective optimization problem rather than a single objective optimization problem that has been considered so far in the literature. Lim et al. [1] depicted the performance of a cyclone, with 10 different vortex finders, to examine the effect of the vortex finder shape on the characteristics of the pressure drop and collection efficiency.

In recent years, there have been many efforts to deploy Genetic Algorithms (GAs) to design artificial neural networks since such evolutionary algorithms are particularly useful for dealing with complex problems having large search spaces with many local optima [2]. In this way, GAs has been used in a feed forward GMDH-type neural network for each neuron searching its optimal set of connection with the preceding layer [3]. In the former reference, authors have proposed a hybrid genetic algorithm for a simplified GMDH-type neural network in which the connection of neurons are restricted to adjacent layers. Moreover a multi-objective genetic algorithm has also been recently used by some of authors to design GMDH-type neural networks considering some conflicting objectives [4].

In this paper, firstly, pressure drop (Δp) and the collection efficiency (η) in a set of cyclones with different vortex finder shape are numerically investigated using FLUENT. Next, genetically optimized GMDH type neural networks are used to obtained polynomial models for the effects of geometrical parameters of the vortex finder on both Δp and η. Such an approach of meta-modeling of those CFD results allows for iterative optimization techniques to design optimally the vortex finder computationally affordable. Finally, the obtained simple polynomial models are used in a Pareto based multi-objective optimization approach to find the best possible combinations of Δp and η, known as the Pareto front.

2. CFD simulation and validation of the results

2.1. CFD simulation

For an incompressible fluid flow, the equation of continuity and balance of momentum are given as:

\[
\frac{\partial u_i}{\partial x_i} = 0 \quad (1)
\]

\[
\frac{\partial \rho u_i}{\partial t} + \frac{\partial \rho u_i u_j}{\partial x_j} = -\frac{1}{\rho} \frac{\partial P}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_i \partial x_j} - \frac{\partial}{\partial x_j} R_{ij} \quad (2)
\]

Where \( R_{ij} = \overline{u_i u_j} \) is the Reynolds stress tensor. Here, \( u_i' = u_i - \overline{u_i} \) is the ith fluctuating velocity component.

The RSTM provides differential transport equations for evaluation of the turbulence stress components where the turbulence production terms are defined as:

\[
\frac{\partial}{\partial x_i} R_{ij} + \frac{\partial}{\partial x_i} B_{ij} = \frac{\partial}{\partial x_i} \left( \frac{v_{ij}}{K} \frac{\partial R_{ij}}{\partial x_i} \right) - \left[ R_{ik} \frac{\partial v_{kj}}{\partial x_i} + R_{jk} \frac{\partial v_{ki}}{\partial x_i} \right] - C_{2} \frac{\varepsilon}{K} R_{ij} - \frac{2}{3} \rho \frac{\partial u_i}{\partial x_i} \left[ P_{ij} - \frac{2}{3} \rho \delta_{ij} \right] - \frac{2}{3} \rho \frac{\partial u_i}{\partial x_i} \quad (3)
\]

\[
P_{ij} = -\left[ R_{ik} \frac{\partial v_{kj}}{\partial x_i} + R_{jk} \frac{\partial v_{ki}}{\partial x_i} \right] \left[ \frac{1}{2} p_{ij} \right] \quad (4)
\]

With \( P \) being the fluctuating kinetic energy production, \( \varepsilon = 1, C_1 = 1.4, C_2 = .6 \) are empirical constants.
The transport equation for the turbulence dissipation rate, $\epsilon$, is given as

$$\frac{\partial \epsilon}{\partial t} + \frac{\partial}{\partial x_i} \left( \nu \frac{\partial \epsilon}{\partial x_i} \right) = \frac{\kappa}{\epsilon} \frac{\partial}{\partial x_i} \left( \nu \frac{\partial \epsilon}{\partial x_i} \right) - C_{\epsilon \epsilon}^{\epsilon \epsilon} \frac{\kappa}{\epsilon} \frac{\partial}{\partial x_i} \left( \frac{\partial \epsilon}{\partial x_i} \right) + \frac{\kappa}{\epsilon} C_{\epsilon \epsilon}^{\epsilon \epsilon} \frac{\partial}{\partial x_i} \left( \frac{\partial \epsilon}{\partial x_i} \right) - C_{\epsilon \epsilon}^{\epsilon \epsilon} \frac{\kappa}{\epsilon} \frac{\partial}{\partial x_i} \left( \frac{\partial \epsilon}{\partial x_i} \right)$$

(5)

In equation (5), $R = \frac{1}{2} u' u_i$ is the fluctuating kinetic energy, and $\epsilon$ is the turbulence dissipation rate. The values of constants are $C_{\epsilon \epsilon}^{\epsilon \epsilon} = 1.44$ and $C_{\epsilon \epsilon}^{\epsilon \epsilon} = 1.92$.

The dispersion of small particles is strongly affected by the instantaneous fluctuation of fluid velocity. The turbulence fluctuations are random functions of space and time. In this study, a discrete random walk (DRW) model is used for evaluating the instantaneous velocity fluctuations. The values of $u'$, $v'$ and $w'$ that prevail during the lifetime of the turbulent eddy, $T_e$ are sampled by assuming that they obey a Gaussian probability distribution. In this model the instantaneous velocity in the $i$th direction is given as

$$u'_i = \frac{\xi_{i} u_i}{\sqrt{\epsilon\delta_{i}}}$$

(6)

In Eq. (6), $\xi$ is a zero-mean, unit-variance, normally distributed random number, $\sqrt{u_i u_i}$ is the local root mean-square (RMS) fluctuation velocity in the $i$th direction, and the summation convention on $i$ is suspended.

The characteristic lifetime of the eddy is defined as a constant given by

$$T_e = 2T_i$$

(7)

Where, $T_i$ is the eddy turn over time given as, $T_i = 3.3 \ k/\epsilon$ in the RSTM. The other option allows for a long --normal random variation of eddy lifetime that is given by

$$T_e = -T_i \ \log r$$

(8)

Where, $r$ is a uniform random number between 0 and 1. The particle is assumed to interact with the fluid fluctuation field, which stays fixed over the eddy lifetime. When the eddy lifetime is reached, a new value of the instantaneous velocity is obtained by introducing a new value of $\xi$ in Eq. (6).

In the present study, one way coupling method is used to solve the two phase flow and the Eulerian-Lagrangian approach is implemented for simulation of second discrete phase (particles). In this model, air is the continuous phase and the particles are treated as the dispersed discrete phase. The volume averaged and steady state Navier-Stokes equations is solved for the gas phase. The particle motions are simulated by the Lagrangian trajectory analysis procedure. Forces acting on the dispersed phase include drag and gravity. The discrete phase equations are solved using Runge-Kutta method for particles.

The particle loading in a cyclone separator is typically small (Raoufi et al., 2008) and therefore, it can be safely assumed that the presence of the particles does not affect the flow field (one-way coupling). In this paper collisions between particles and the walls of the cyclone were assumed to be perfectly elastic (Gimbin et al., 2005; Safikhani et al., 2010). Also, particle-particle collision is negligible.

Inlet mass flow boundary condition is used at the cyclone inlet and a fully developed boundary condition is used at the outlet. The computation is continued until the solution converged with a total residual of less than 0.00001.

2.2. Definition of the design variables

The design variables in present paper are: the angle of vortex finder ($\theta$), upside length of vortex finder ($L_{up}$), downside length of vortex finder ($L_{down}$) and diameter of vortex finder ($D_v$). It should be noted that $L_{up}$ plus $L_{down}$ is vortex finder depth in the cyclone body (yellow part in Figure 1). The design variables are shown in Figure 2. By
changing the geometrical independent parameters, various designs will be generated and evaluated by CFD. Consequently, some meta-models can be optimally constructed using the GMDH-type neural networks, which will be further used for multi-objective Pareto based design of such cyclones. In this way, 81 various CFD analyses have been performed due to those different design geometrics.

![Diagram](image)

**Figure 2:** Definition of design variables in a divergent (a) and convergent (b) vortex finder

### 3. Modeling of pressure drop and collection efficiency using GMDH type neural networks

By means of GMDH algorithm a model can be represented as set of neurons in which different pairs of them in each layer are connected through a quadratic polynomial and thus produce new neurons in the next layer. Such representation can be used in modeling to map inputs to outputs. The formal definition of the identification problem is to find a function \( \hat{f} \) so that can be approximately used instead of actual one, \( f \) in order to predict output \( \hat{y} \) for a given input vector \( X = (x_1, x_2, x_3, ..., x_n) \) as close as possible to its actual output \( y \). Therefore, given \( M \) observation of multi-input-single-output data pairs so that

\[
y_i = f(x_{1i}, x_{2i}, x_{3i}, ..., x_{ni}) \quad (i = 1, 2, ..., M)
\]

It is now possible to train a GMDH-type neural network to predict the output values \( \hat{y}_i \) for any given input vector \( X = (x_{i1}, x_{i2}, x_{i3}, ..., x_{in}) \), that is

\[
\hat{y}_i = \hat{f}(x_{i1}, x_{i2}, x_{i3}, ..., x_{in}) \quad (i = 1, 2, ..., M)
\]

The problem is now to determine a GMDH-type neural network so that the square of difference between the actual output and the predicted one is minimized, that is

\[
\sum_{i=1}^{M} [f(x_{i1}, x_{i2}, x_{i3}, ..., x_{in}) - y_i]^2 \rightarrow \min
\]

General connection between inputs and output variables can be expressed by a complicated discrete form of the Volterra functional series in the form of
\[ y = a_0 + \sum_{i=1}^{n} a_i x_i + \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} x_i x_j + \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{n} a_{ijk} x_i x_j x_k + \ldots \]  

(12)

Where is known as the Kolmogorov-Gabor polynomial (Farlow, 1984). This full form of mathematical description can be represented by a system of partial quadratic polynomials consisting of only two variables (neurons) in the form of

\[ \hat{y} = G(x_i, x_j) = a_0 + a_1 x_i + a_2 x_j + a_3 x_i x_j + a_4 x_i^2 + a_5 x_j^2 \]

(13)

There are two main concepts involved within GMDH-type neural networks design, namely, the parametric and the structural identification problems. In this way, some works by Jamali et al. (2009), present a hybrid GA and singular value decomposition (SVD) method to optimally design such polynomial neural networks. The methodology in these references has been successfully used in this paper to obtain the polynomial models of the \( \Delta p \) and \( \eta \). The obtained GMDH-type polynomial models have shown very good prediction ability of unforeseen data pairs during the training process which will be presented in the following sections.

The input–output data pairs used in such modeling involve two different data tables obtained from CFD simulation discussed in Section 2. Both of the tables consist of four variables as inputs, namely \( \theta, D_e, L_{up} \) and \( L_{Down} \) (Figure 2) and outputs, which are \( \Delta p \) and \( \eta \). The tables consist of a total of 81 patterns, which have been obtained from the numerical solutions to train and test such GMDH type neural networks. However, in order to demonstrate the prediction ability of the evolved GMDH type neural networks, the data in both input–output data tables have been divided into two different sets, namely, training and testing sets. The training set, which consists of 61 out of the 81 input–output data pairs for \( \Delta p \) and \( \eta \), is used for training the neural network models using the method presented in Section 2. The testing set, which consists of 20 unforeseen input–output data samples for \( \Delta p \) and \( \eta \) during the training process, is merely used for testing to show the prediction ability of such evolved GMDH type neural network models. The GMDH type neural networks are now used for such input–output data to find the polynomial models of \( \Delta p \) and \( \eta \) with respect to their effective input parameters. In order to design genetically such GMDH type neural networks described in the previous section, a population of 10 individuals with a crossover probability (\( P_c \)) of 0.7 and mutation probability (\( P_m \)) 0.07 has been used in 500 generations for \( \Delta p \) and \( \eta \). The corresponding polynomial representation for \( \Delta p \) is as follows:

\[ Y_1 = 0.04735 + 0.299 D_e + 0.3541 L_{up} - 0.01662 D_e^2 - 0.00761 L_{up}^2 - 0.0129 D_e L_{up} \]

(14.a)

\[ Y_2 = 0.244 - 0.0561 \theta + 0.915 L_{Down} - 0.00331 \theta - 0.04112 L_{Down}^2 - 0.00110 L_{Down} \theta \]

(14.b)

\[ Y_3 = 0.042 + 0.32 L_{up} + 0.160 L_{Down} - 0.007 L_{up}^2 - 0.004 L_{Down}^2 + 0.0007 L_{up} L_{Down} \]

(14.c)

\[ Y_4 = 1.2810 - 0.0905 \theta + 0.8140 D_e - 0.003311 \theta^2 - 0.03733 D_e^2 + 0.001786 D_e \theta \]

(14.d)

\[ Y_5 = 0.23121 + 0.01000 Y_1 - 0.13081 Y_2 - 0.82e - 12 Y_1^2 + 0.01761 Y_2^2 + 0.24710 Y_1 Y_2 \]

(14.e)

\[ Y_6 = 0.2682 - 0.02e - 11 Y_3 - 0.140 Y_4 + 3.68e - 012 Y_3^2 + 0.0179 Y_4^2 + 0.2500 Y_3 Y_4 \]

(14.f)

\[ \Delta P = 0.00651 + 0.9285 Y_5 + 0.06777 Y_6 - 0.03120 Y_3^2 + 0.1075 Y_6^2 - 0.07532 Y_3 Y_6 \]

(14.g)

Similarly, the corresponding polynomial representation of the model for \( \eta \) is in the form of

\[ Y_1' = 0.4362 + 2.763 D_e + 3.2711 L_{up} - 0.13442 D_e^2 - 0.06823 L_{up}^2 - 0.00918 D_e L_{up} \]

(15.a)
The good behavior of such GMDH type neural network model for pressure drop and efficiency is also depicted in Figure 3, both for the training and testing data. It is evident that the evolved GMDH type neural network in terms of simple polynomial equations successfully model and predict the outputs of the testing data that have not been used during the training process. The models obtained in this section can now be utilized in a Pareto multi-objective optimization of the vortex finder considering both pressure drop and efficiency as conflicting objectives. Such study may unveil some interesting and important optimal design principles that would not have been obtained without the use of a multi-objective optimization approach.

\[ Y_2' = 2.031 - 0.24015\theta + 7.604L_{Down} - .0025\theta - 0.2921L_{Down}^2 - .016633L_{Down}\theta \]  
\[ Y_3' = 0.37 + 2.78L_{up} + 1.392L_{Down} - .063L_{up}^2 + 0.0016L_{Down}^2 + .01751L_{up}L_{Down} \]  
\[ Y_4' = 1.1481 - 0.702\theta + 7.27981D_e - .0025331\theta^2 - 0.313222D_e^2 + .0228211D_e\theta \]  
\[ Y_5' = 2.072 - .0499Y_1' - .049Y_2' - 4.732e-11Y_1'^2 - 1.01261e-5Y_2'^2 + .0251Y_1'.Y_2' \]  
\[ Y_6' = -2.026 - 2.316e-9Y_3' + 0.096Y_4' + 1.132e - 011Y_3'^2 - 0.0011Y_4'^2 + .023Y_3'.Y_4' \]  
\[ \eta = 0.19312 - 1.1879Y_3' + 2.17961Y_6' - .50361Y_5'^2 + .528411Y_6'^2 + 1.03241Y_3'.Y_6' \]  

Figure 3: CFD vs. Network

4. Multi-objective optimization of vortex finder using polynomial neural network models

In order to investigate the optimal performance of the vortex finder in different geometrical parameters the polynomial neural network models obtained in section 3 are now deployed in a multi-objective optimization procedure. The two conflicting objectives in this study are \( \Delta p \) and \( \eta \) that are to be simultaneously optimized with respect to the design variables \( \theta, D_e, L_{up} \) and \( L_{Down} \). The 2-objective optimization problem can be formulated in the following form:
Maximize Collection Efficiency \( (\eta) = f_1(\theta, D_e, L_{up}, L_{Down}) \)

Minimize Pressure Drop \( (\Delta p) = f_2(\theta, D_e, L_{up}, L_{Down}) \)

Subject to \(-10^o < x_1 < 10^o\)
\[ 11 < x_2 = D_e < 15 \]
\[ 10 < x_3 = L_{up} < 30 \]
\[ 5 < x_4 = L_{Down} < 15 \] (16)

The evolutionary process of Pareto multi-objective optimization is accomplished by using the modified NSGA-II approach (Jamali et al., 2009) where a population size of 1024 and a generation number of 300 has been chosen in different runs with crossover probability \( P_c \) and mutation probability \( P_m \) are 0.9 and 0.01 respectively.

Figure 4 depicts the obtained non-dominated optimum design points as a Pareto front of those two objective functions. There are five optimum design points, namely, A, B, C, D, and E whose corresponding design variables and objective functions are shown in Table 1. These points clearly demonstrate tradeoffs in objective functions pressure drop and collection efficiency from which an appropriate design can be compromise-ly chosen. It is clear from Figure 4 that all the optimum design points in the Pareto front are non-dominated and could be chosen by a designer as optimum vortex finder. Evidently, choosing a better value for any objective function in the Pareto front would cause a worse value for another objective. The corresponding decision variables of the Pareto front shown in Figure 4 are the best possible design points so that if any other set of decision variables is chosen, the corresponding values of the pair of objectives will locate a point inferior to this Pareto front. Such inferior area in the space of the two objectives is in fact bottom/right side of Figure 4.

![Figure 4: Multi-objective Pareto result for cyclone vortex finders](image)

In Figure 4, the design points A and E stand for the best pressure drop and the best collection efficiency. Moreover, the other optimum design points, B and D can be simply recognized from Figure 4. The design point, B exhibit important optimal design concepts. In fact, optimum design point B obtained in this paper exhibits an increase in pressure drop (about 15.5%) in comparison with that of point A whilst its efficiency improves about
33.6% in comparison with that of $A$, similarly optimum design point $D$ exhibits a decrease in efficiency (about 29.5%) in comparison with that of point $E$ whilst its pressure drop improves about 48.6% in comparison with that of $E$. It is now desired to find a trade-off optimum design points compromising both objective functions. This can be achieved by the method employed in this paper, namely, the mapping method. In this method, the values of objective functions of all non-dominated points are mapped into interval $0$ and $1$. Using the sum of these values for each non-dominated point, the trade-off point simply is one having the minimum sum of those values. Consequently, optimum design point $C$ is the trade-off points which have been obtained from the mapping method.

**Table 1:** The values of objective functions and their associated design variables of the optimum points

<table>
<thead>
<tr>
<th>Points</th>
<th>$\theta$ (deg)</th>
<th>$D_x$ (mm)</th>
<th>$L_{up}$ (mm)</th>
<th>$L_{Down}$ (mm)</th>
<th>$\Delta p$ (cm water)</th>
<th>$\eta$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>10</td>
<td>14.873</td>
<td>10</td>
<td>5</td>
<td>2.434</td>
<td>25.724</td>
</tr>
<tr>
<td>$B$</td>
<td>10</td>
<td>15</td>
<td>10</td>
<td>15</td>
<td>3.062</td>
<td>39.171</td>
</tr>
<tr>
<td>$C$</td>
<td>10</td>
<td>13.920</td>
<td>27.142</td>
<td>15</td>
<td>4.106</td>
<td>51.545</td>
</tr>
<tr>
<td>$D$</td>
<td>9.603</td>
<td>12.396</td>
<td>25.238</td>
<td>15</td>
<td>4.545</td>
<td>53.931</td>
</tr>
<tr>
<td>$E$</td>
<td>-10</td>
<td>11.126</td>
<td>22.380</td>
<td>15</td>
<td>6.495</td>
<td>65.714</td>
</tr>
</tbody>
</table>

The Pareto front obtained from the GMDH-type neural network model (Figure 4) has been superimposed with the corresponding CFD simulation results in Figure 5. It can be clearly seen from this figure that such obtained Pareto front lies on the best possible combination of the objective values of CFD data, which demonstrate the effectiveness of this paper, both in deriving the model and in obtaining the Pareto front.

![Figure 5: Overlay graph of the obtained optimal Pareto front with the numerical data.](image-url)
5. Conclusion

Genetic algorithms have been successfully used both for optimal design of generalized GMDH type neural networks models of pressure drop and efficiency in cyclones and for multi-objective Pareto based optimization of vortex finder. Two different polynomial relations for pressure drop and efficiency have been found by evolved GS-GMDH type neural networks using some numerically validated CFD simulations for input–output data of the vortex finder. The derived polynomial models have been then used in an evolutionary multi-objective Pareto based optimization process so that some interesting and informative optimum design aspects have been revealed for cyclones with respect to the control variables of vortex finder geometrical parameters namely $\theta$, $D_v$, $L_{up}$ and $L_{down}$ (Figure 2). Consequently, some very important facts in the optimum design of vortex finder have been obtained and proposed based on the Pareto front of two conflicting objective functions. Such combined application of GMDH type neural network modeling of input–output data and subsequent non-dominated Pareto optimization process of the obtained models is very promising in discovering useful and interesting design relationships.

References