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Reduced Order Modelling for Reliability Optimisation of Advanced Micro-Systems

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Abstract
This paper discusses the Design for Reliability of advanced electronics Micro-systems based on computational approach that integrates methods for high fidelity analysis, reduced order modelling, numerical risk analysis and optimisation. The methodology is demonstrated for the design of a System-in-Package (SiP) structure. System-in-Package is a technology that is developed on the basis of miniaturised integrated multi-functional electronics modules using 3D stacking of several silicon chips (Integrated Circuits, ICs). System-in-Package aims to provide fully functional electronic systems and sub-systems that integrate several functionally different devices, e.g. optical, MEMS, sensors and other components, into a single package. There is little understanding and knowledge how do the large die sizes in the SiP modules, the lead-free assembly, interfacial de-laminations and the utilisation of new materials affect the reliability of these electronics systems. In particular, the board level reliability of the package related to the thermal fatigue material degradation of solder interconnects is of a great concern. Understanding the performance, reliability and robustness of SiP modules is a key factor for the future development and success of the technology.

The main focus in this study is on the techniques for reduced order modelling and the development of the associated models for fast design evaluation and analysis. The discussion is on methods for approximate response surface modelling based on interpolation techniques using Kriging and radial basis functions. The reduced order modelling approach uses prediction data for the thermo-mechanical behaviour of the SiP design obtained through non-linear transient finite element simulations, in particular for the fatigue life-time of the lead-free solder interconnects and the warpage of the package.

The reduced order models are used for the analysis of the effect of design uncertainties on the reliability of these advanced electronics modules. To aid this assessment, different methods for estimating the variation of reliability related metrics of the electronic package are researched and tested. Sample based methods such as full scale Monte Carlo and Latin Hypercube, and analytical approximate methods such as First Order Second Moment (FOSM) and Point Estimation Method (PEM) are investigated and their accuracy is compared.

The optimisation modelling addresses the probabilistic nature of the reliably problem of the SiP structures under investigation. Optimisation tasks with design uncertainty are formulated and solved using modified Particle Swarm Optimisation algorithms. The probabilistic optimisation deals with two different performance metrics of the design, the thermo-mechanical fatigue reliability of the board level interconnects and the thermally induced warpage of the package. The objective in this analysis is to ensure that the design has the required reliability and meets a number of additional requirements.

Keywords: Reduced Order Models, System-in-Package, Risk Analysis, Probabilistic Optimisation, Microsystems.

1. Introduction

The three-dimensional micro integration design concept of the SiP structures and the increased package functionality combined with shorter design development cycles is resulting in a decreased knowledge about the performance, reliability and robustness of these electronic modules [1]. Key advantages of the SiP technological concept range from reduction in package complexity and size to lower cost and design effort. A particular issue of concern is how performance and reliability aspects of the systems might be influenced and to what degree by the uncertainties associated with package design parameters. Having the right tools and strategies to support the design development is critical requirement for the success of the technology. The goal is to ensure that a package design will provide the required reliability despite the degree of performance variation due to the uncertainty of the design inputs and their propagation into the actual response characteristics.

Simulation based optimisation for virtual design prototyping of various electronic packages and manufacturing processes has proven as an effective approach for process characterisation and product development at the early design stages [2-4]. Normally, these strategies are used to obtain the deterministic optimal package design based on the variation of a number of input parameters so that imposed constraints and design requirements are satisfied. However, in reality such optimal package design, from deterministic point of view, may be far from a reliable, safe and robust design solution. The reason for this is in the uncertainties in various design and process parameters. It is very difficult and often impossible to control such existing variations. These tolerances and variations of the input design parameters may have significant impact on the system behaviour and can lead to variations and scatter of the response parameters that define the target requirements for performance and reliability. In order to provide the required reliability of the designed system, the uncertainties associated with the input parameters must be taken into account. From an optimisation pint of view, the aim is to identify a design solution that always meets the design constraints despite of the existing variations in the system/process response parameters.

This paper focus the discussions on three key aspects of the Design for Reliability methodology: (1) the development of reduced order models for fast analysis evaluations using Finite Element Analysis, (2) modelling of the uncertainties of the SiP design inputs and responses, and (3) optimal SiP design identification through reliability-driven probabilistic numerical optimisation.

2. Computational Analysis of SiP Thermo-Mechanical Behaviour and Reliability

A System-in Package structure that is built upon stacking of two silicon dies is investigated. The active die in this module is flipped onto the passive die. The board level solder joints are designed in two peripheral rows along each side of the passive die as illustrated in Figure 1. This SiP component is then placed on a printed circuit board (PCB). To improve the thermo-mechanical reliability of the board level solder joints, underfill material is used to fill the gap between the PCB and the passive die.

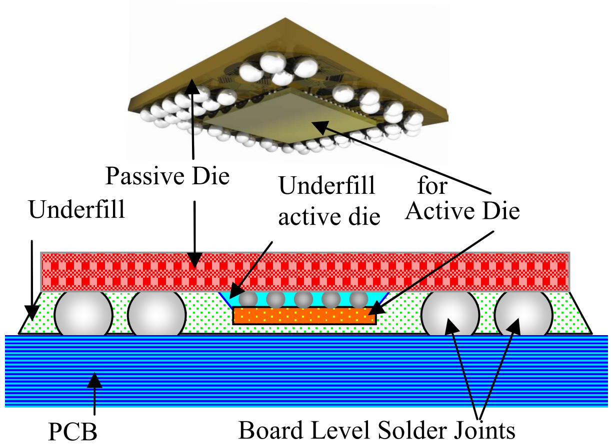

Figure 1: System-in-Package structure

Table 1 lists some of the key assembly dimensions of interest. The second column specifies the geometry of the nominal (or initial) design of the SiP while the third column of the table provides details on some possible design variations of the SiP assembly parameters that are feasible to implement.

Table 1: Material properties

System-in-Package Design Variables	Nominal Values / mm	Un-scaled Limits / mm	Scaled Limits / dimensionless
PCB Thickness	1.00	0.80 to 1.20	-1 to 1
Stand-off Height of Solder Joints	0.235	0.21 to 0.26	-1 to 1
Passive Die Thickness	0.20	0.15 to 0.25	-1 to 1

As detailed in Table 1, the following SiP design parameters (design variables) can be varied from their nominal values:

1. PCB thickness (h_{PCB});
2. Board level solder joints stand-off-height (h_{SOH});
3. Passive die thickness (h_{DIE}).

By changing the value of any of these design variables, design modifications of the SiP structure can be generated. A set of values for the specified design variables that specify a particular design is referred as a *design point*.

Due to the existing symmetry in the SiP structure, it is sufficient to represent in the computer model for finite element analysis only one-eight part of the assembly. The finite element model which captures 1/8 of package is shown in Figure 2.

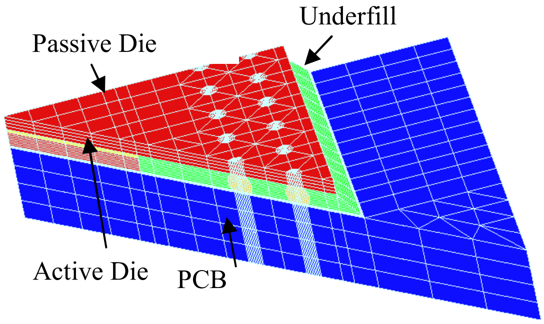

Figure 2: Finite Element Analysis model of the System-in-Package

The thermo-mechanical response of the SiP structure is analysed under accelerated thermal cycling from -40 °C to 125 °C. Time-dependent plasticity and creep accompanied by stress relaxation for the lead-free solder joints are modelled using inelastic strain rate *sinh* constitutive law (Eq. 1):

$$\dot{\varepsilon}_{ij}^{cr} = A(sinh(\alpha\,\sigma_{eff}))^{n}\ exp\left(-\frac{Q}{R\,T}\right) \tag{1}$$

where $\dot{\varepsilon}_{ij}^{cr}$ is inelastic strain rate, R is the gas constant, T is the temperature in Kelvin, σ_{eff} is the effective (Von Mises) stress, and all other symbols represent material related constants for Sn3.9Ag0.6Cu solder.
Inelastic strain and stress in solder joints and package deformations are predicted from the non-linear FEA. These response values are used to calculate the damage in solder joints in terms of accumulated inelastic energy density per thermal cycle *Wp*. The solder joint life prediction model (Eq. 2) is used then to correlate the damage *Wp* to life-time in terms of cycles to failure [5]:

$$N_f = (0.0014\,Wp)^{-1} \tag{2}$$

Figure 3 illustrates a typical prediction results for the system response to temperature cycle load from finite element analysis. The contours of the inelastic energy density across the solder joints and the package deformation used to calculate the warpage parameter in Figure 3 refer to the design with for the following SiP specification: (h_{PCB} = 0.8 mm, h_{SOH} = 0.21 mm, h_{DIE} = 15 mm).

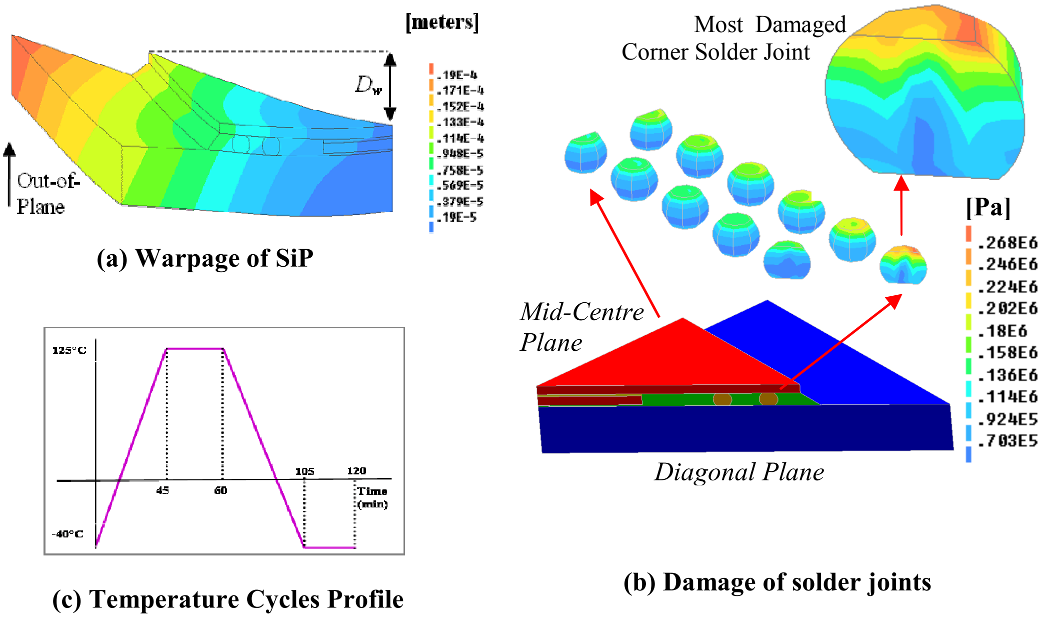

Figure 3: Contour levels for (a) out-of-plane deformation of the package at 125°C, and (b) inelastic work density across SiP solder joints (initial design) at the end of a thermal cycle (c)

The results from the finite element simulations show that the most critical solder joint (i.e. likely to fails first) is always the one located at the corner of the package. The non-linear FEA is used also predict the deformations across the SiP assembly. A response of interest is the maximum warpage of the SiP during the thermal cycling. This quantity is defined as the difference between the minimum and maximum out-of-plane deflection of the package and is denoted as D_w. The maximum warpage occurs at the highest temperature during the thermal cycle (125°C).

Full details on the SiP structure and materials, and the modelling approach for thermo-mechanical reliability analysis can be found in reference [6].

3. Reduced Order Modelling through Data Interpolation Approach

Reduced order modelling (ROM) involves the formulation of models that can evaluate and analyse a system or process behaviour quickly and without significant computational effort and time. A reduced order model is typically an approximation of the true behaviour of the system. The models are very useful to observe design trends and issues, especially at the early design stages of a product development. The reduced order modelling strategy presented here is based on so-called response surface generation and uses data obtained through DoE methods. The models are developed as functions of the design variables for investigation. In this work the discussions are on two interpolation-based models, Kriging and Radial basis functions. The models are demonstrated for the System-in Package described previously.

3.1. Design of Experiments

The first step in the ROM approach is to derive performance data through DoE by evaluation of a limited number of design scenarios. From a design point of view, any design modification of the System-in-Package is restricted to changing the PCB thickness (h_{PCB}), the stand-off height of solder joints (h_{SOH}) and the passive die thickness (h_{DIE}). DoE methods are applied in the three-dimensional design space of the design defined by the respective limits for the PCB thickness (0.8 mm to 1.2 mm), the stand-off height of solder joints (0.21 mm to 0.26 mm) and the passive die thickness (0.15 mm to 0.25 mm). A Central Composite Design method is applied to this design space to provide fifteen design points. The fifteen experimental points include the 8 factorial and the 6 axial points, and the central point of the defined design space. The DoE points are listed in Table 2. The table also shows the dimensionless scaled values of design variables over the range -1 to 1 used in the subsequent development of the reduced order models. The last two columns list the finite element analysis predictions for life-time and maximum wearpage in the package for each of the DoE points in the table.

Table 2: DoE points and predicted System-in-Package responses from FEA

DoE Point No	PCB thickness /mm		Stand-off height of solder joints /mm		Passive die thickness /mm		Cycles to Failure *Nf*	Maximum Warpage *Dw*
	Actual	Scaled	Actual	Scaled	Actual	Scaled		
1	0.8	-1	0.21	-1	0.15	-1	2990	11.92
2	1.2	1	0.26	1	0.25	1	2255	7.52
3	0.8	-1	0.26	1	0.25	1	2780	10.75
4	1.2	1	0.21	-1	0.15	-1	2409	7.93
5	0.8	-1	0.21	-1	0.25	1	2809	11.55
6	1.2	1	0.26	1	0.15	-1	2437	7.35
7	0.8	-1	0.26	1	0.15	-1	2973	11.05
8	1.2	1	0.21	-1	0.25	1	2232	8.1
9	1	0	0.235	0	0.15	-1	2659	9.23
10	1	0	0.235	0	0.25	1	2480	9.24
11	1	0	0.21	-1	0.2	0	2537	9.68
12	1	0	0.26	1	0.2	0	2542	8.99
13	0.8	-1	0.235	0	0.2	0	2877	11.38
14	1.2	1	0.235	0	0.2	0	2319	7.77
15	1	0	0.235	0	0.2	0	2548	9.31

3.2. Kriging Reduced Order Models

Kriging reduced order modelling is an interpolation technique used to predict unknown values from data observed at known points. It minimises the error at the predicted values that are estimated from the distribution of the observed data. The Kriging model is defined as.

$$y(X) = \sum_{j=0}^{m} \beta_j P_j(X) + \sum_{i=1}^{n} \alpha_i \varphi(h_i) \tag{3}$$

where X is the vector of the m design variables, $X = (x_1, ..., x_m)$ and β_j (j = 0,...,m) are the coefficients of the polynomials $P_j(X)$ (j = 0,...,m). In equation (3), α_i (i = 1,...,n) are the coefficients of the basis functions $\varphi(h_i)$ (i = 1,...,n) where n is the number of the DoE points. The polynomials $P_j(X)$ in this case are linear (i.e. $P_j(X) = x_j$, j = 1,...,m and $P_0(X) = 1$).

The basis function $\varphi(h_i)$ is called a variogram and has, as argument, the absolute distance between points X and X_i. There are many variogram models available. The spherical model is utilised in this study as it is suitable for cases with a small number of design variables, typically when m ≤ 3 [7]. The spherical model is defined a

$$\varphi(h_i) = \begin{cases} 0 & \text{if} \quad h_i = 0 \\ C_1\left(\dfrac{1.5h_i}{C_2} - 0.5\dfrac{h_i^3}{C_2^3}\right) & \text{if} \quad 0 < h_i \le C_2 \\ C_1 & \text{if} \quad h_i > C_2 \end{cases} \tag{4}$$

where $h_i = \|X - X_i\|$ (i = 1,...,n), and C_1 and C_2 are the variogram coefficients. The unknown coefficients in the Kriging ROMs, β_j ($j = 0,...,m$), α_i ($i = 1,...,n$), C_1 and C_2 are computed so that the error of variation of the predicted and observed data is minimised [7]. The Kriging model for the mean fatigue life of solder joints F_{Nf} using the fifteen DoE points in Table 2 is defined as

$$\begin{aligned} &F_{\mathrm{Nf}}(h_{\mathrm{PCB}}, h_{\mathrm{SOH}}, h_{\mathrm{DIE}}) = 2600 - 277.64\, h_{\mathrm{PCB}} + \\ &0.857\, h_{\mathrm{SOH}} - 91.36\, h_{\mathrm{DIE}} + 2510.9 \sum_{i=1}^{n} \alpha_i^{Nf} \left(\frac{1.5h_i}{2.041} - \frac{0.5h_i^3}{4.166}\right) \\ &h_i = \sqrt{\left(h_{\mathrm{PCB}} - (h_{\mathrm{PCB}})_i\right)^2 + \left(h_{\mathrm{SOH}} - (h_{\mathrm{SOH}})_i\right)^2 + \left(h_{\mathrm{DIE}} - (h_{\mathrm{DIE}})_i\right)^2} \end{aligned} \tag{5}$$

The Kriging model for the maximum warpage of the package F_{Dw} using the fifteen DoE points in Table 2 is defined as

$$\begin{aligned} &F_{\mathrm{Dw}}(h_{\mathrm{PCB}}, h_{\mathrm{SOH}}, h_{\mathrm{DIE}}) = 9.469 - 1.798\, h_{\mathrm{PCB}} - \\ &0.352\, h_{\mathrm{SOH}} - 0.033\, h_{\mathrm{DIE}} + 0.058 \sum_{i=1}^{n} \alpha_i^{Dw} \left(\frac{1.5h_i}{1.704} - \frac{0.5h_i^3}{2.904}\right) \\ &h_i = \sqrt{\left(h_{\mathrm{PCB}} - (h_{\mathrm{PCB}})_i\right)^2 + \left(h_{\mathrm{SOH}} - (h_{\mathrm{SOH}})_i\right)^2 + \left(h_{\mathrm{DIE}} - (h_{\mathrm{DIE}})_i\right)^2} \end{aligned} \tag{6}$$

where $i = 1,...,n$ ($n = 15$, number of DoE points) and the values of the coefficients α_i^{Nf}, α_i^{Dw} are listed in Table 3. When constructing Kriging ROMs, it is possible to evaluate the prediction capability of the model by computing the bandwidth, or the error interval, of the ROM predicted value at a particular design point. Figure 4 shows the bandwidths of the warpage Kriging ROM for each the three design variables. In each of the three respective plots the values of the other two design variables are kept at their nominal values (as listed in Table 1). The prediction interval plots in Figure 4 are derived as 95% confidence bandwidths for the predicted values and are based on the assumption that the warpage response follows a normal distribution.

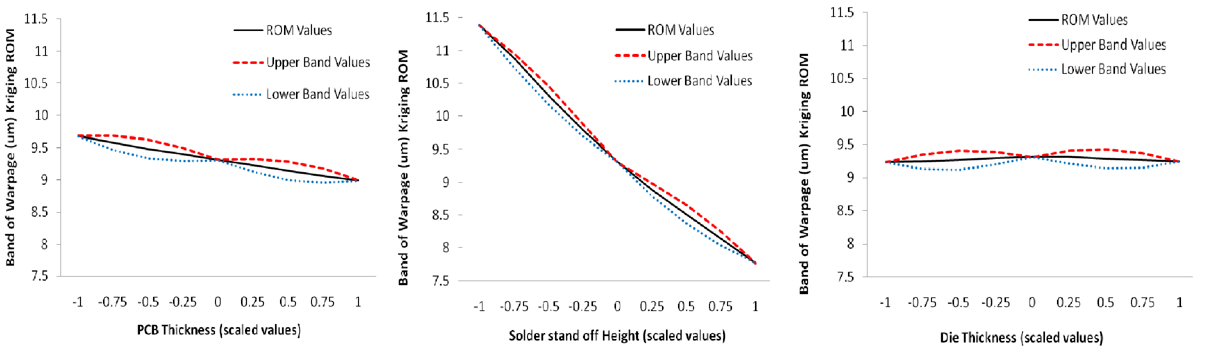

Figure 4: Bandwidth for warpage Kriging model at the nominal design of the SiP

Table 3: Coefficients for Kriging and Radial Basis function reduced order models.

System-in-Package DoE scaled values of design variable			Coefficients for reduced order models			
			Kriging		Radial Basis	
$(h_{\mathrm{PCB}})_i$	$(h_{\mathrm{SOH}})_i$	$(h_{\mathrm{DIE}})_i$	α_i^{Nf}	α_i^{Dw}	γ_i^{Nf}	γ_i^{Dw}
-1	-1	-1	-0.0128	-4.84	16230	615.4
1	1	1	-0.013	-4.19	6253	-434.5
-1	1	1	-0.0008	2.17	-58750	-413.9
1	-1	-1	-0.0026	2	62660	342.7
-1	-1	1	-0.0133	0.50	10520	-524
1	1	-1	-0.0129	-0.15	12990	-525.8
-1	1	-1	-0.0051	-1.95	7830	133.3
1	-1	1	-0.0047	-2.04	1383	-48.57
0	0	-1	0.0110	4.69	-75800	-1918
0	0	1	0.0093	3.32	63790	1121
0	-1	0	0.0269	2.25	-144100	-430.9
0	1	0	0.0254	1.99	-20930	143.9
-1	0	0	-0.0057	-2.55	120100	756
1	0	0	-0.0044	-2.29	13300	181.2
0	0	0	0.0027	1.12	-14720	-49.49

3.3. Radial Basis Reduced Order Models

Radial basis models are also types of functions that pass through all the DoE data points. In this case, the model formulation is based on a radial basis function $\varphi(\|X - X_i\|)$ and linear polynomials $P_j(X)$ ($j = 0,\dots,m$).

$$y(X) = \sum_{j=0}^{m} \beta_j P_j(X) + \sum_{i=1}^{n} \gamma_i r(h_i) \tag{7}$$

where X, β_j and $P_j(X)$ denote the same entities as defined in Section 3.2, γ_i are the coefficients in the radial basis function, $r(h_i)$, and h_i is the Euclidean distance between point X and a DoE point X_i ($h_i = \|X - X_i\|$, $i = 1,\dots,n$) [8,9]. Several classes of radial basis function may be chosen for $r(h)$ [9]. In this study we use the Gaussian radial basis function $r(h) = e^{-0.01h^2}$. The radial basis model for the mean fatigue life of solder joints R_{Nf} is

$$R_{\mathrm{Nf}}(h_{\mathrm{PCB}}, h_{\mathrm{SOH}}, h_{\mathrm{DIE}}) = 3839.42 - 237.6\,h_{\mathrm{PCB}} - 45.32\,h_{\mathrm{SOH}} - 143.7\,h_{\mathrm{DIE}} + \sum_{i=1}^{n} \gamma_i^{Nf} e^{-0.01 h_i^2} \tag{8}$$

The radial basis model for the maximum warpage of the package R_{Dw} is

$$R_{\mathrm{Dw}}\left(h_{\mathrm{PCB}}, h_{\mathrm{SOH}}, h_{\mathrm{DIE}}\right) = 13.65 - 1.582\,h_{\mathrm{PCB}} - 0.568\,h_{\mathrm{SOH}} - 1.174\,h_{\mathrm{DIE}} + \sum_{i=1}^{n} \gamma_i^{Dw} e^{-0.01 h_i^2} \tag{9}$$

where $i = 1,\ldots,n$ ($n = 15$ in the example), h_i has the same definition as in equation (3) and the values of the coefficients γ_i^{Nf}, γ_i^{Dw} are listed in Table 3. All design variables are scaled between -1 and 1.

The studies presented in the following sections are based on the use of the radial basis ROMs above as it is slightly more computationally efficient to derive them compared with the Kriging models.

3. Risk Analysis

In the case of miniaturised integrated electronics products and systems, it is important to gain knowledge about the effect of design and manufacturing uncertainties on performance and to assess if key product characteristics fall outside tolerable specification limits. This would typically require evaluation of the actual variation distribution of the performance parameters. The distributions can then be used for the purpose of capability calculations and risk mitigation.

The techniques for probabilistic distribution estimation discussed here involve both sampling based and analytical methods. The sampling methods investigated are the Monte Carlo and the Latin Hypercube methods, and the analytical methods are the Mean Value First Order Second Moment (FOSM) and the Point Estimation Method (PEM). The Monte Carlo sampling is based on random sampling of each design variable using its probability distribution thus generating different design samples. Typically, a very large number (thousands or millions) of design samples must be generated. Latin Hypercube Sampling (LHS) is a sampling scheme based on a technique known as "stratified sampling without replacements" [10]. The FOSM method is based on a first order Taylor series approximation of the reduced order model linearised at the mean values of the design variables [11]. PEM is an approximation method, which does not require knowledge of the particular form of probability distribution of the design variables. PEM is a weighted average method of numerical integration formula involving sampling points and weighting parameters [11].

To demonstrate these techniques, we assume the PCB thickness, solder joint height and silicon die thickness follow normal Gaussian distributions with standard deviations of 16 μm, 2 μm and 2.5 μm respectively. The predictions for the cycles to failure and warpage distribution from Monte Carlo, Latin Hypercube, FOSM and PEM methods, for the nominal design case of the system-in package ($h_{\mathrm{PCB}} = 1$ mm, $h_{\mathrm{SOH}} = 0.235$ mm and $h_{\mathrm{DIE}} = 0.2$ mm) are shown in Figure 5. The values for the SiP responses of interest are obtained using the Radial basis reduced order models. The numerical results are also given in Tables 4 and 5. It was found that in this case the analytical and the Latin Hypercube methods provide very close estimates for the mean value (9.312 μm) and the standard deviation (0.1472 μm) of warpage compared with the prediction from a large scale Monte Carlo simulation. Similarly, hypercube methods provide very close estimates for the mean value (2548 cycles to failure) and the standard deviation (23 cycles to failure) of mean fatigue life of solder joints compared with large scale Monte Carlo simulation.

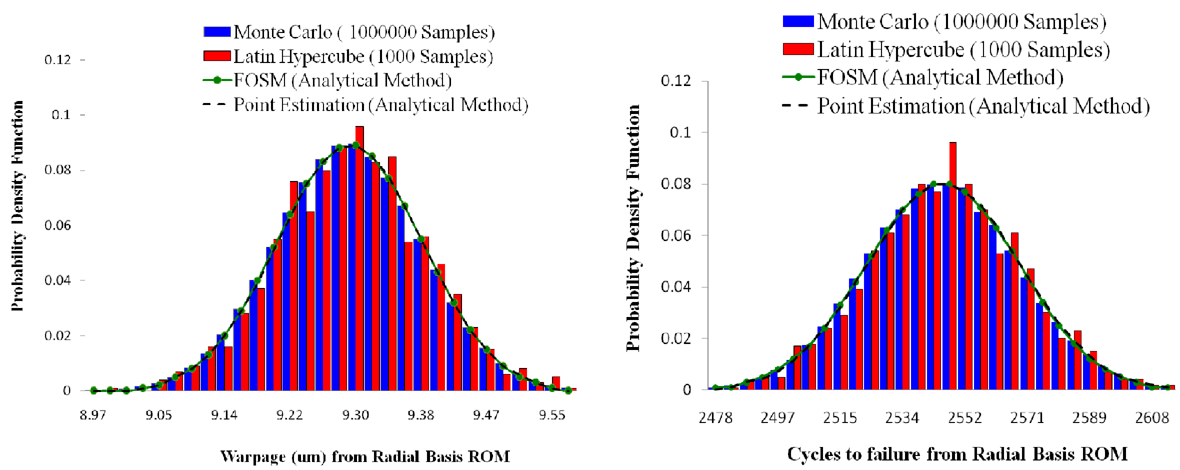

Figure 5: Distribution of warpage (left) and cycles to failure (right) obtained by various distribution estimation methods at the nominal design of the System-in-Package.

Table 4: Warpage distribution first and second moments by various distribution estimation methods.

	MCS (1 Million Samples)	LHS (1000 Samples)	FOSM	PEM
Mean (Warpage / um)	9.312	9.312	9.310	9.312
Standard Deviation	0.1472	0.1482	0.1472	0.1472

Table 5: Cycles to failure distribution first and second moments by various distribution estimation methods.

	MCS (1 Million Samples)	LHS (1000 Samples)	FOSM	PEM
Mean (cycles)	2548	2548	2548	2548
Standard Deviation	22.86	22.82	22.79	22.79

In the case of the SiP nominal design, product capability calculations are demonstrated below. The product capability analysis can use any of the above distribution estimator methods and different capability metrics. As a demonstration, the Cpk rate of capability is derived. The value of Cpk is the difference between the parameter mean value and the nearest specification limit divided by three times the standard deviation. For example, if the fatigue life of solder joints must be at least 2450 cycles to failure, then this will be a lower specification limit (LSL) for the reliability of this particular design (i.e. LSL = 2450 cycles to failure). In this case we consider the SiP design capability with respect to the fatigue life reliability and therefore no upper specification limit is required. A risk analysis based on the respective radial basis ROM and a Monte Carlo simulation (1 million points) is undertaken to obtain the uncertainty distribution of the fatigue life of solder joints. In this case it can be predicted that will be no failures and all packages will meet the specification limit requirement (Figure 5-a). The capability metric is Cpk = 1.43 (i.e. >1) implying the design is capable of complying with the specified reliability requirement. However, if we consider the same design with different lower specification limit, 2500 cycles to failure, similar analysis reveals that 1.54% of the packages fall outside the specification limit and Cpk =0.70. As Cpk is less than one, it can be concluded that under the specification limit LSL= 2500 cycles to failure the initial SiP design is not capable to satisfy reliability requirement.

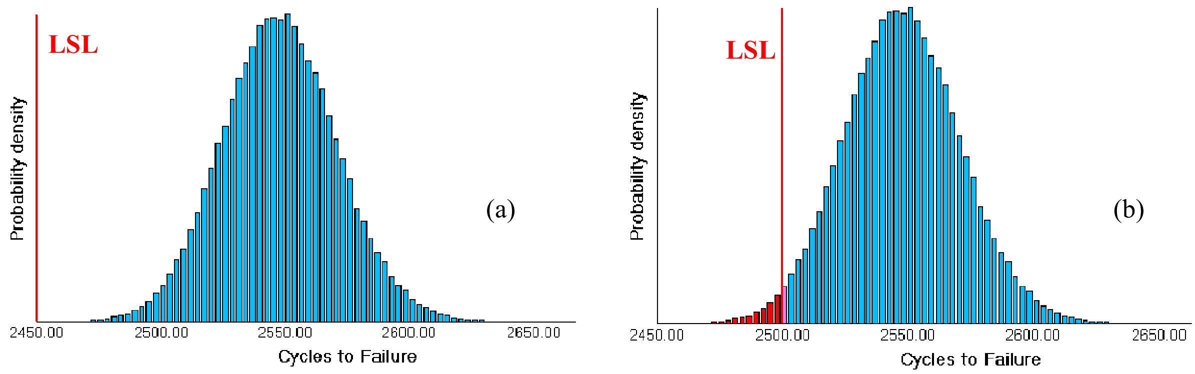

Figure 5: Distribution of mean fatigue life of solder joint under specified lower limit for reliability (a) LSL = 2450 cycles to failure and (b) LSL = 2500 cycles to failure for nominal SiP design.

4. System-in-Package Design Optimisation

The design optimisation task is to identify SiP structure for which the warpage of the package is minimised while satisfying a life-time requirement of 2700 cycles to failure. An additional constraint which requires the total thickness of the SiP package to be less than or equal to 400 microns is also imposed. The design modifications are restricted to changes of the three design variables with the design variable limits specified in Table 1. In this study, a design for the SiP is defined as *reliable* if it satisfies all defined constraints. Normally the distribution of the probabilistic input design variables is known and can be specified through certain distribution parameters. In this study the variation is defined by Gaussian distribution and uses two parameters, the mean value and the standard deviation. The following standard deviations specify the uncertainty of the SiP design variables:

a) h_{PCB}: standard deviation σh_{PCB} = 16 μm;
b) h_{SOH}: standard deviation σh_{SOH} = 2 μm;
c) h_{DIE}: standard deviation σh_{DIE} = 2.5 μm;

The uncertainty properties of the responses are usually unknown. Therefore, when uncertainties are included in the design optimisation task, we need to estimate the random properties of the responses. As shown in the previous section, different methods can be used to obtain this information. One way is to calculate the response mean value and standard deviation and to use this information to judge the probability of failure with respect to that response. Monte Carlo Simulation is the technique used in this study although if the computational time is an issue the analytical methods should be exploited.
In reliability based optimisation the aim is to account for the variations of the responses that define the reliable design domain and to ensure that the deterministic optimal solution is moved from the boundary of the active constraints inside the feasible domain. Therefore, the aim is to minimise or satisfy constraints that involve system responses and the related probability of failure. This reliable optimum design is called a probabilistic, or reliable, optimum. To define the probabilistic optimum one must specify what probability of failure will be acceptable.
To demonstrate the reliability based design optimisation strategy, the following formulation of the design problem above is given:

Find values of h_{PCB}*,* h_{SOH} *and* h_{DIE} *that*
Minimise Warpage of SiP, D_w
Subject to:
(c1) P(Life-time $N_f \leq 2\,700$) ≤0.05
(c2) P($h_{SOH}+h_{DIE} \geq 0.40$ mm) ≤0.05
(c3) $0.8 \leq h_{PCB} \leq 1.2$ mm
Standard deviation σh_{PCB} = 16 μm
(c4) $0.21 \leq h_{SOH} \leq 0.26$ mm
Standard deviation σh_{SOH} = 2 μm
(c5) $0.15 \leq h_{DIE} \leq 0.25$ mm
Standard deviation σh_{DIE} = 2.5 μm

The solution of this optimisation problem will account for the variation of the input design variables (the constraints (c3)-(c5)). The constraint (c1) states that the probability of the fatigue life being less than or equal to 2700 cycles to failure must be no greater than 0.05 (i.e. 5 % probability of failure limit with respect to the life-time requirement). Similarly, the constraint (c2) is formulated to represent a reliability requirement on the package thickness, i.e. the probability of SiP thickness ($h_{SOH}+h_{DIE}$) becoming greater than or equal to 400 microns must be no greater than 0.05.
The above optimisation problem is defined and solved using ROMARA [12]. The optimisation task is solved using both a gradient based algorithm and a particle swarm optimisation (PSO) algorithm. Both techniques identify the same optimal solution (within the algorithm tolerances). Design evaluations during optimisation are based on the radial basis ROMs. The numerical optimisation solution procedure incorporates Monte Carlo simulations at each of the design optimisation iterations in order to evaluate the probabilities of failure as defined in (c1) and (c2). The solution of the design for reliability problem is reported in Table 6. The probabilistic optimum design, despite the uncertainty of the input parameters, is formulated so that it is 95 % reliable with respect to design constraints (c1) and (c2).

Table 6: Reliable probabilistic optimum

	Probabilistic Reliable Optimum
Optimal h_{PCB} [mm]	0.947
Optimal h_{SOH} [mm]	0.241
Optimal h_{DIE} [mm]	0.150
Warpage D_w [μm]	9.68
Life-time mean value N_f [cycles]	2 738
Standard deviation of Life-time N_f [cycles]	25
P(Life-time $N_f \leq$ limit)	0.05
SiP thickness: $h_{SOH}+h_{DIE}$ [mm]	0.391
Standard deviation of ($h_{SOH}+h_{DIE}$) [mm]	0.003
P($h_{SOH}+h_{DIE} \geq 0.40$ mm)	0.001

5. Conclusions

This paper has outlined a Design for Reliability methodology. The application of the reliability-based design optimisation is demonstrated for the area of advanced multi-functional electronics packages. A methodology for developing fast design evaluation models using high fidelity simulation and Design of Experiments methods is presented. Response surface based reduced order models that can capture the non linear behaviour are presented. A study of several different distribution estimator techniques is conducted to identify a more efficient approach to derive the variation of a performance parameter than a conventional Monte Carlo simulation. The methods are integrated for the purpose of carrying out reliability based design optimisation. The optimal solution of the analysed System-in-Package product with respect to reliability has been found in a very efficient and automated way using both gradient based and non-gradient numerical techniques.

Future work will focus on multi-objective probabilistic optimisation where both reliability an robustness of the product will be considered. The complexity of the design task is also planned to increase as we intend to introduce problems where the product may be subjected to different failure mechanism and failure modes.

References

[1] K.L. Tai, System-In-Package (SIP): Challenges and Opportunities, *Proceedings of Asia and South Pacific Design Automation Conference (ASP-DAC'00)*, Yokohama, Japan, 191-196, 2000.

[2] G. N. Vanderplaats, *Numerical Optimisation Techniques for Engineering Design: with Applications*, VR&D, Colorado Springs , USA, 1999.

[3] G.Q. Zhang, P. Maessen, J. Bisschop, J. Janssen, F. Kuper and L. Ernst, Virtual Thermo-Mechanical prototyping of Microelectronics – the Challenges for Mechanics Professionals", *Proceedings of EuroSIME*, 2001, pp. 21-24.

[4] S. Stoyanov, *Optimisation Modelling for Microelectronics Packaging and Product Design*, PhD Thesis, University of Greenwich, London, UK, 2004.

[5] A. Syed, Accumulated Creep Strain and Energy Density Based Thermal Fatigue Life Prediction Models for SnAgCu Solder Joints, *Proceedings of the 54-th ECTC*, Las Vegas, USA, June 2004, pp. 737-746.

[6] N. Strusevich, S. Stoyanov, D. Liu, C. Bailey, A. Richardson, N. Dumas, J.M. Yannou and V. Georgel, "Modelling the Behavior of Solder Joints for Wafer Level SiP", *Proceedings of the 8th EPTC*, Singapore, 6-8 December 2006, pp. 127-132.

[7] N. Cressie, *Statistics for Spatial Data*, John Wiley & Sons, New York, 1991.

[8] N. Gumerov and R. Duraiswami, Fast radial basis function interpolation via preconditioned Krylov iteration, *SIAM. J. Sci. COMPUT*, Vol. 29, No. 5, 2007, 1876-1899.

[9] J. G. Wang and G. R. Liu, A point interpolation meshless method based on radial basis function, *Int. J. Numer. Meth. Engng*, Vol. 54, 2002, 1623-1648.

[10] K. Ding, Z. Zhou and C. Liu, Latin hypercube sampling used in the calculation of fracture probability, *Reliability Engineering and System Safety*, Vol. 59, 1998, 239-242.

[11] D. Vose, *Risk Analysis, A Quantitative Guide*, Third edition, John Wiley & Sons, 2008.

[12] ROAMARA, Reduced Order Modelling and Risk Analysis software, University of Greenwich, London, UK, http:// cmrg.gre.ac.uk/software/ROMARA