# Application of a maintenance model for optimizing tamping on ballasted tracks: the influence of the model constraints

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#### Abstract

This paper presents an application of a developed model for scheduling tamping on ballasted tracks that takes into account the evolution over time of the track degradation, the track's layout, the dependency of track quality recovery on the quality at the moment of maintenance operations and also the track quality limits that depend on train speed. All these aspects are considered in a mathematical programming through suitable constraints formulated as a nonlinear mixed problem. In the present work, the influence of track layout and quality recovery in scheduling tamping is analysed for a ballasted railway track designed for a speed of 220 km/h.

**Keywords:** Railway track; preventive maintenance; tamping; scheduling; integer mixed problem; linear programming.

## 1. Introduction

Maintenance actions are of fundamental importance for the safe and efficient operation of trains on a railway track and also for passenger comfort. Track maintenance covers all the measures for preserving and re-establishing the track nominal condition. When performing track maintenance, tamping is the measure usually adopted to correct the longitudinal level, which is the geometrical parameter that most influences vehicles and track dynamics in the vertical direction. That is why the longitudinal level is the geometrical parameter considered in this maintenance model. Preventive maintenance increase not only the system reliability but also its availability, but the resulting costs are also increased, which may be minimized by scheduling maintenance operations, such as tamping, trough optimization programs.

Optimization problems may be solved by exact algorithms; heuristics and meta-heuristics algorithms (genetic models); hybrid algorithms and multi-objective algorithms [1]. These algorithms have already been applied for optimizing railway timetables, assigning locomotive at a minimal operational costs, optimizing networks, assigning extra trains on a railway network, etc [2, 3, 4, 5, 6].

The application of mathematical programming for scheduling preventive maintenance of railway tracks is relatively new, however there are already some contributions on this theme by different types of optimization models. Higgins et al. [7] develop a model to help solving the conflicts between train operations and the scheduling of maintenance activities and its formulation is based on integer programming. The model is then applied to a 89 km track corridor on the eastern coast of Australia considering a four day planning horizon. The authors conclude that their model could be used by local track managers and train planners in real-time if integrated into a train dispatching real-time database. That way, adjustments could be made to a planned schedule of activities. Budai et al. [8] present a preventive maintenance model where a schedule for the maintenance activities has to be found for one link by minimizing the sum of the possession costs and the maintenance costs. The possession costs are the cost associated to the time that a track requires for maintenance and cannot be used for railway traffic. The authors focus on the medium-term planning, determining which preventive maintenance work will be performed in what time periods (month/week/hours). Oyama et al. [9] present a mathematical programming model for the optimal decision-making for tamping that consists in two steps: 1) a transition model for predicting changes in the surface irregularity; 2) a mathematical programming model to define the optimal maintenance strategies for the annual tamping schedule. The tamping is done with a Multiple Tie Tamper (MTT) machine that is shared by several track maintenance depots. The model indicates not only which month the MTT should be allocated to a particular depot but also which track segment with 100 m of lenght should be provided with maintenance work. Zhao et al. [10] define a model to optimize sleeper maintenance by minimizing the number of sleepers to be replaced to meeting the requirements of reliability and safe sleeper operation. To obtain the near-optimal solution of the problem, the authors use the steepest gradient method.

# 2. Briefly description of the maintenance model

A maintenance model was developed for scheduling tamping on ballasted tracks in order to take into account the evolution over time of the track degradation, the track's layout, the dependency of track quality recovery on track quality at the moment of maintenance operations and also the track quality limits that depend on train speed. In this paper a briefly description of the model is presented; the complete model formulation may be seen in [11].

Some assumptions were considered in the model formulation:

- a) the degradation of track geometry is characterized by the increase of the standard deviation of the longitudinal level, which means that irregularities of alignment, cross level, gauge and twist are disregarded for now constraint (2);
- b) maintenance actions (binary variable) correspond to tamping operations (constraint 6) and are performed in order to the standard deviation of the longitudinal level to be always inferior to its corresponding limit constraint (5);
- c) the recovery of track quality is linearly dependent on the value of the standard deviation of the longitudinal level of the track segment at the time of maintenance because it has been reckoned that the recovery effectiveness of the longitudinal level depends on the quality of the track at the time of maintenance operations [12] constraint (3);
- d) tamping operations begin and end on a straight alignment, according to UIC [13] recommendations - constraint (4).

The maintenance model called (P1) seeks an optimal solution corresponding to the minimum of the total number of tamping actions M on a track for a predefined time horizon. (P1)

$$M = \min \sum_{i=1}^{n_t} \sum_{j=1}^{n_p} m_{ij}$$
(1)

subject to

$$\sigma_{ij} = \sigma_{ij-1} + d_i - m_{ij}r_{ij} \tag{2}$$

$$r_{ij} = a(\sigma_{ij-1} + d_i) + b \tag{3}$$

$$\sum_{l \in I(k)} m_{lj} \ge |I(k)| \, m_{kj} \tag{4}$$

$$0 \le \sigma_{ij} \le \sigma_{lim} \tag{5}$$

$$m_{ij} \in \{0, 1\} \tag{6}$$

with  $i \in \{1, ..., n_t\}$ ,  $k \in \{1, ..., n_t\}$  and  $j \in \{1, ..., n_p\}$ . The model parameters already presented above are:

- .  $n_t$ , total number of track segments;
- .  $n_p$ , total number of discrete time periods (time horizon);
- .  $d_i$ , degradation rate of the longitudinal level on segment track i;
- .  $\sigma_{i0}$ , standard deviation of the longitudinal profile at an initial time (j = 0);

- .  $\sigma_{lim}$ , limit for the standard deviation of the longitudinal profile depending on vehicle speed;
- .  $a \in b$ , real parameters;
- . I(k), set of consecutive indexes of track segments that include segment k in curve and an initial and final segments in straight alignment;
- . |I(k)|, number of elements of I(k).

The problem variables, that total  $3n_t n_p$ , are:

- .  $\sigma_{ij}$ , standard deviation of the longitudinal level of track segment i and time period j;
- .  $r_{ij}$ , recovery after maintenance of the standard deviation of the track longitudinal level on segment i and time period j;
- .  $m_{ij}$ , binary variable that denotes whether maintenance activity is assigned to track segment *i* and time period *j* ( $m_{ij} = 1$ ) or not ( $m_{ij} = 0$ )

The optimization problem described by equations (2) to (6) consists in minimizing a linear function in binary variables on a set of linear and bilinear constraints. Therefore the mathematical model is a nonlinear mixed binary problem. The bilinear terms  $m_{ij}r_{ij}$  can be transformed into variables by exploiting the so-called Reformulation-Linearization Technique (RLT) [14, 15, 16]. These transformations lead into a linear mixed binary program as described in [11].

# 3. Model application: the influence of model constraints

The model application is performed with Cplex solver of the Gams software [17]. The data adopted for the calculations are:

- the rail track is composed by 250 segments of 200 m of lenght;
- the initial standard deviation of each track segment varies between 0.1 and 0.6 mm (new track), as shown in Figure 1;

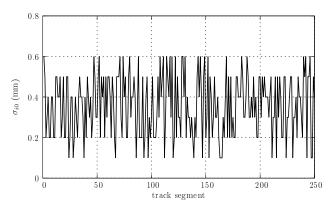


Figure 1: Initial standard deviation of each track segment

- the time horizon corresponds to eight time instants of 90 days, which is the usual time interval between track inspections;
- the degradation rate of the standard deviation of the longitudinal level is constant over time: 0.1712 mm in 90 days;
- the track layout is composed by straight alignments (R) and curves (C), as presented in Figure 2
- the limit value for the standard deviation of the longitudinal level is 1.5 mm, corresponding to the alert limit for 220 km/h, according to EN13848-5 [18].

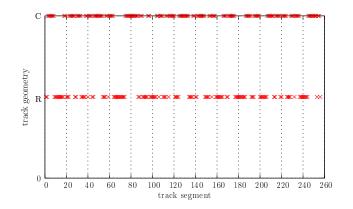


Figure 2: Track layout

Four distinct scenarios are analyzed in order to study the influence of track layout and track quality recovery on scheduling tamping for ballasted railway tracks. These scenarios are:

- CC1 . no consideration of track layout and the recovery of the standard deviation of the longitudinal level is a constant value (r = 0.4855 mm);
- CC2 . consideration of the track layout and the recovery of the standard deviation of the longitudinal level is a constant value (r = 0.4855 mm);
- CC3 . no consideration of track layout and the recovery of the standard deviation of the longitudinal level depends linearly on the track quality at the moment of tamping, as presented in Equation (7); this expression was defined based on real records measured in a Portuguese Railway Line.

$$r_{ij} = 0.4257(\sigma_{i,j-1} + d_i) - 0.153 \le r_{max} \tag{7}$$

CC4 - Complete model

. consideration of both the track layout and the linear dependency of the recovery of the standard deviation of the longitudinal level on the track quality at the moment of tamping.

To disregard the track layout constraint in the model application (scenarios CC1 and CC3) is the same to consider that the track stretch in study is a straight alignment. The track layout adopted for scenarios CC2 and CC4 is representing in Figure 2: 52.4% of the total number of segment tracks are in curve.

Before showing the results, it is reported that the choice of a value of 0.4855 mm for the recovery of the standard deviation of the longitudinal level (scenarios CC1 and CC2) is made in order to compare the results obtained in these two scenarios, with the attained in scenarios CC3 and CC4. Note that in scenarios CC3 and CC4, the recovery of the track quality is limited, implicitly, to 0.4855 mm, as is shown below. That occurs because the recovery of the standard deviation of the longitudinal level depends on the longitudinal level at the time of maintenance, which is at the maximum 1.5 mm - the alert limit for a train speed of 200 km/h.

$$r_{max} = 0.4257 \times 1.5 - 0.153 = 0.4855 \text{ mm}$$
(8)

In Table 1, the results attained for each one of the four scenario are shown.

Comparing scenarios CC1 with CC2 and also CC3 with CC4, which differ in regard to the consideration whether or not of the track layout, it appears that the distribution of curve segments on the track stretch in study, increases the total number of shares maintenance at approximately 7% for the studied situation. This increase of maintenance actions depends not only on the percentage of the segments in curve, but also on its distribution along the railway stretch.

In terms of the total number of maintenance actions, the results obtained with the application of the model show that for the time period under analysis (two years), in scenarios CC2 and CC4, only a single maintenance action is expected for all track segments excepting segments 24, 67, 120, 215 and 230. For scenarios CC1 and CC3, it is also scheduled only a single action for maintenance for all the track segments, except on 22 of them, for which there are no predicted maintenance actions for the two years time horizon.

Scenario	Total number of	Total CPU
	maintenance actions	time (s)
CC1	228	0.158
CC2	245	0.316
CC3	228	0.442
CC4	245	0.312

Table 1: Results

The results presented in Table 1 indicate that the optimal number of maintenance actions obtained in the calculation is the same for scenarios CC3 and CC1 and also for CC2 and CC4. The (only) difference between these pair of scenarios is the consideration or not of the constraints referring to the linear dependence of the recovery of the standard deviation of the longitudinal level on the track quality when maintenance is performed. Note that, because the limitation of the standard deviation of longitudinal level to 1.5 mm, also the recovery of that standard deviation after tamping is limited (in the scenarios CC3 and CC4) to 0.4855 mm as indicated in Equation (8). This value is the one adopted for the constant recovery of the standard deviation of the longitudinal level in scenarios CC1 and CC2. Therefore the total number of maintenance actions in scenarios CC3 and CC4, tends to the number in scenarios CC1 and CC2 because a global optimization approach is being performed.

However, there may be some differences in the distribution of maintenance actions over time, as a result of considering the dependence of the recovery of the standard deviation of the longitudinal level on the track quality when tamping is performed. In Figure 3, the distribution over time of maintenance actions for CC2 and CC4 is presented.

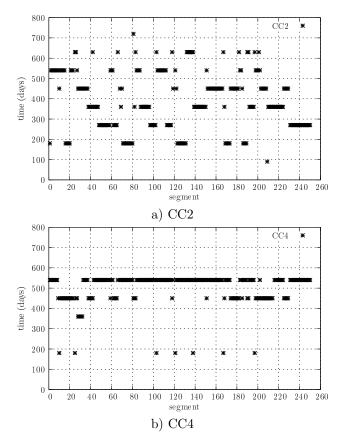
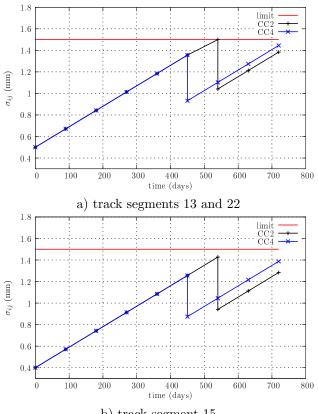


Figure 3: Distribution over time of maintenance actions

The reason for this difference lies in the fact that in scenario CC2, the track recovery has always a constant value, while in scenario CC4, the recovery of the standard deviation of the longitudinal level

depends on the standard deviation value at the moment maintenance action is done.

In Figure 4, the evolution of the standard deviation of the longitudinal level predicted by scenario CC2 and CC4 are presented for track segments 13, 22 and 15.



b) track segment 15

Figure 4: Evolution of the standard deviation of the longitudinal level

The Figure shows that, in these three track segments, only a single maintenance action is scheduled during the two years time period, as already mentioned. This Figure also shows that the quality of the segments, after 720 days in CC2 scenario is slightly better than the one obtained in scenario CC4, as a consequence of performing the maintenance action in a subsequent moment in comparison with CC4.

Still about the influence of the model constraints (3) a (6), if at scenarios CC1 and CC2, the constant value for the recovery of the standard deviation of the longitudinal level is less than 0.0486 mm, it is no longer possible to compare these scenarios with CC3 and CC4. In this case, the total number of maintenance actions increases in relation to the solution attained with CC3 or CC4, because as the value of the maximum recovery is higher in these scenarios in relation to scenarios CC1 or CC2 (with r = 0.0486mm), it is possible to ensure the quality of track geometry with a smaller number of maintenance actions.

In Table 2, the results obtained for scenario CC2 with r = 0.3 mm and for CC4 with recovery dependency defined by equation (7) prove the comment above referred.

Table 2: Results: influence of the recovery value of the longitudinal level after tamping

Scenario	$r_{ij}$	Total numeber of	Total CPU
	(mm)	maintenance actions	time (s)
CC2	0.3	404	0.312
CC4	$0.4257(\sigma_{i,j-1} + d_i) - 0.153$	245	0.312

#### 4. Conclusions

An application of a developed maintenance model for scheduling tamping on ballasted tracks has been presented in this paper. The model is formulated as a nonlinear mixed binary problem and takes into account the evolution over time of the track degradation, the track's layout, the dependency of quality recovery on track quality at the moment of maintenance operations and also the quality limits that depend on train speed.

The model application allows to make some considerations about the influence of track layout and track quality recovery on scheduling tamping. The results show that the consideration of track layout in scheduling tamping operations increase the total number of (predicted) maintenance actions. This increase depends not only on the percentage of the segments in curve, but also on its distribution along the railway stretch. It is also reckoned that the linear dependency of the recovery of the standard deviation of the longitudinal level on the track quality at the moment of tamping operations influences results. As far as the model is concerned, this maintenance model is able to produce useful results in terms of

optimal schedules in a reasonable time for the test application shown here.

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