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Finding the elastic coefficients of a damaged zone in a concrete dam using material optimization to fit measured modal parameters

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Abstract
Given the knowledge of the first natural frequencies, the material coefficients in a cracked zone of a dam are identified through an inverse problem. The main ingredient are parametric optimization, the derivatives of the eigenvalues of the structure and gradient methods. An algorithm and numerical results will accomplish the study.
Keywords: Free Material Optimization, Derivatives of the Eigenfrequencies, Inverse Problems.

1. Introduction

The structural safety control of large dams is based on the continuous observation of its behavior under static and dynamic actions and on the use of mathematical models for the simulation of the real dam behavior. These mathematical models should be calibrated taking into account the real behavior of the system dam-reservoir-foundation (mainly influenced by the reservoir water level). For the safety control of old dams, usually with cracking, the mathematical models should include some features for crack simulation.

2. Motivation

In this paper the dynamic behavior of a cracked arch dam - Cabril dam - is studied by means an elastic 3D FE model that was calibrated using an optimization technique for the computation of the main elastic coefficients that must be used for the cracked zone in order to fit the experimentally measured natural frequencies. Changes on modal parameters are associated not only to water level variations but could be also correlated with structural changes (due to accidental loads, as strong earthquakes, or deterioration processes along the time - e.g. deterioration associated with the development of concrete swelling). For this reason the identification of these changes may be interesting for the structural safety control. The Cabril dam is the highest concrete dam in Portugal, with 130 meters of height and crest length of 290 meters. It is an interesting case study because a large number of cracks have arisen in a specific part of the dam since the beginning of its operations in 1954 (see Figure 1). These cracks have remained even after reparation works done in 1981 (see [1]).
The free vibrations of the dam are being continuously measured by a dynamic monitoring system installed in 2008 (see [2]). The values of the measured natural frequencies are used in our approach. Since the cracks are mainly disposed in the horizontal direction, a transversely isotropic constitutive law is chosen to model the damaged zone of the dam (in red in Figure 2). The rest of the dam (in yellow in Figure 2) is modeled using an isotropic constitutive law. The transversely isotropic constitutive law

$$\begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{bmatrix} = \begin{bmatrix} \frac{1}{E_p} & \frac{-\nu_p}{E_p} & \frac{-\nu_{zp}}{E_z} & 0 & 0 & 0 \\ \frac{-\nu_p}{E_p} & \frac{1}{E_p} & \frac{-\nu_{zp}}{E_z} & 0 & 0 & 0 \\ \frac{-\nu_{zp}}{E_z} & \frac{-\nu_{zp}}{E_z} & \frac{1}{E_z} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{2(1+\nu_p)}{E_p} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{zp}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{zp}} \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{bmatrix} \tag{1}$$

is characterized by five independent elastic constants: E_p, ν_p, E_z, ν_{zp} and G_{zp}. For E_p and ν_p the standard values of the concrete can be used, as done for the isotropic constitutive law of the undamaged part of the dam. The other three constants E_z, ν_{zp} and G_{zp} remain unknown.

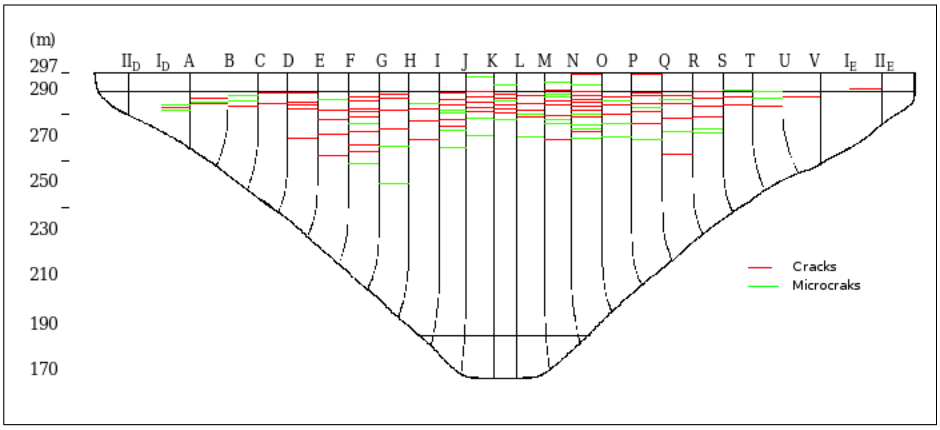

Figure 1: Cabril dam: outline with the location of cracks

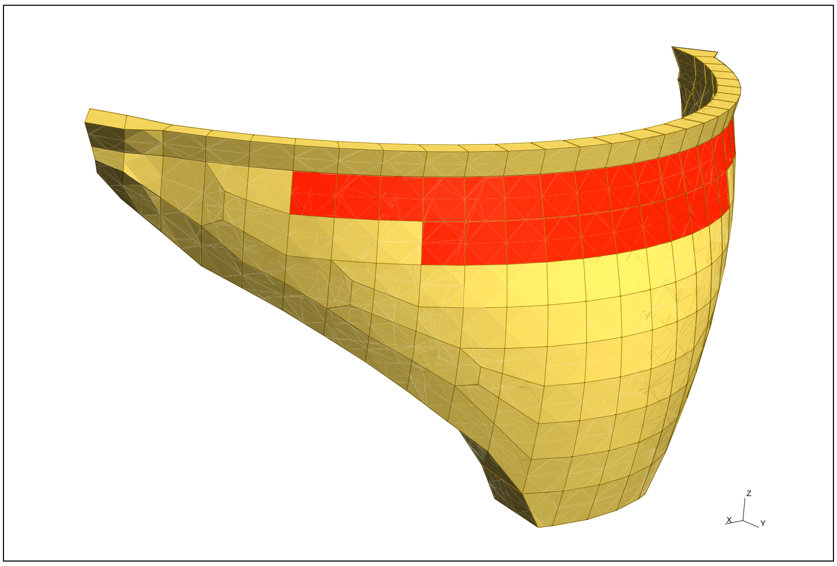

Figure 2: Cabril dam: mesh used in the simulations (in red is the subdomain relative to the damaged zone)

The goal of the present work is to find an approximation of these three values by solving an inverse problem: given the knowledge of the first natural frequencies of the dam, we will search for elastic coefficients such that the solutions of the corresponding eigenvalue problem are the closest to the given frequencies. This approach is related to the field of free material optimization, where the elastic moduli are the variables to be optimized to satisfy some functional cost (see section 3 of [4]).

3. The model problem

The eigenvalue problem below is considered

$$\begin{cases} -div\big(\boldsymbol{C}\epsilon(u)\big) = \lambda\, u\,, & \text{in the dam,} \\ u = 0\,, & \text{on the fixed boundary,} \\ \boldsymbol{C}\epsilon(u)\,.\,n = 0\,, & \text{on the free boundary,} \end{cases} \tag{2}$$

where $\boldsymbol{C}$ is the fourth-order elasticity tensor. Its solutions $(\lambda_k, u_k)_{k\geq 1}$ depend on the five elastic parameters characterizing the transversely isotropic law according to (1). The eigenvalues and eigenfrequencies of this problem are respectively the natural frequencies and modes of vibration of the body in consideration. We consider the following parametric optimization problem:

$$\min_{E_z,\nu_{zp},G_{zp}} J(E_z, \nu_{zp}, G_{zp}) \tag{3}$$

where

$$J(E_z, \nu_{zp}, G_{zp}) = \sum_{i=1}^{N} |\lambda_i(E_z, \nu_{zp}, G_{zp}) - \bar{\lambda}_i|^\alpha \tag{4}$$

for some $\alpha > 0$, where $\lambda_i(E_z, \nu_{zp}, G_{zp})$ represents the first N eigenvalues of the problem (2) (see section 4). Therefore the derivatives of the natural frequencies λ_i with respect to the optimization parameters E_z, ν_{zp} and G_{zp} are needed. The result below gives the derivative of λ_i with respect to a parameter s.

Theorem (see [3]). *Provided differentiability properties of the elasticity tensor $\boldsymbol{C} = \boldsymbol{C}(s)$ with respect to a general parameter s and assuming that the eigenvalues of the problem (2) are simple, the eigenvalue $\lambda_i = \lambda_i(s)$ is differentiable with respect to s and the following formula holds:*

$$\frac{d\lambda_i}{ds} = \int_\Omega \frac{d\boldsymbol{C}}{ds} \epsilon(u_i) \cdot \epsilon(u_i)\, dx, \tag{5}$$

where the corresponding eigenmode u_i is normalized in $L^2(\Omega)$: $\|u_i\| = 1$.

Consequence. *In our case the above integral is taken only on the subdomain Ω_D corresponding to the damaged zone, which is where we allow the parameters to change (in the rest of the subdomain the parameters are fixed). Therefore,*

$$\frac{d\lambda_i}{ds} = \int_{\Omega_D} \frac{d\boldsymbol{C}}{ds}\, \epsilon(u_i) \cdot \epsilon(u_i)\, dx\,.$$

This allow one to obtain the derivative of the functional defined in (4) with respect to a general parameter s, as

$$\frac{dJ}{ds} = \alpha \sum_{i=1}^{N} \frac{|\lambda_i - \bar{\lambda}_i|^\alpha}{\lambda_i - \bar{\lambda}_i} \frac{d\lambda_i}{ds}.$$

In particular the gradient of J is recovered with the formula below

$$\begin{aligned}
\nabla J(E_z, \nu_{zp}, G_{zp}) &= \alpha \sum_{i=1}^{N} \frac{|\lambda_i - \bar{\lambda}_i|^\alpha}{\lambda_i - \bar{\lambda}_i} \left(\frac{\partial \lambda_i}{\partial E_z}, \frac{\partial \lambda_i}{\partial \nu_{zp}}, \frac{\partial \lambda_i}{\partial G_{zp}} \right) \\
&= \alpha \sum_{i=1}^{N} \frac{|\lambda_i - \bar{\lambda}_i|^\alpha}{\lambda_i - \bar{\lambda}_i} \times \\
&\left(\int_{\Omega_D} \frac{\partial \boldsymbol{C}}{\partial E_z} \epsilon(u_i) \cdot \epsilon(u_i) dx,\ \int_{\Omega_D} \frac{\partial \boldsymbol{C}}{\partial \nu_{zp}} \epsilon(u_i) \cdot \epsilon(u_i) dx,\ \int_{\Omega_D} \frac{\partial \boldsymbol{C}}{\partial G_{zp}} \epsilon(u_i) \cdot \epsilon(u_i) dx \right).
\end{aligned}$$

4. Numerical implementation and results

The whole simulation is implemented in C++ using libMesh open-source framework (see [5]). The description of the algorithm is as follows:

- give the first N natural frequencies (physically measured) $\bar{\lambda}_1, \bar{\lambda}_2, \ldots, \bar{\lambda}_N$ and an initial guess of the material parameters in the damaged zone Ω_D, $(E_z^{(0)}, \nu_{zp}^{(0)}, G_{zp}^{(0)})$;
- repeat for $k = 0, 1, 2, \ldots$ until convergence:
  1. Compute the eigenfrequencies $\lambda_1, \lambda_2, \ldots, \lambda_N$ and the corresponding eigenmodes $u_1, u_2, \ldots, u_N$, solutions of problem (2) with $\boldsymbol{C} = \boldsymbol{C}(E_z^{(k)}, \nu_{zp}^{(k)}, G_{zp}^{(k)})$;
  2. Compute $\nabla J(E_z^{(k)}, \nu_{zp}^{(k)}, G_{zp}^{(k)})$, the gradient of the functional J;
  3. Compute a descent direction $d^{(k)}$;
  4. Update the parameters, $(E_z^{(k+1)}, \nu_{zp}^{(k+1)}, G_{zp}^{(k+1)}) = (E_z^{(k)}, \nu_{zp}^{(k)}, G_{zp}^{(k)}) + \gamma^{(k)}\, d^{(k)}$, where $\gamma^{(k)}$ is a step that is computed in order to ensure that the functional J decreases;

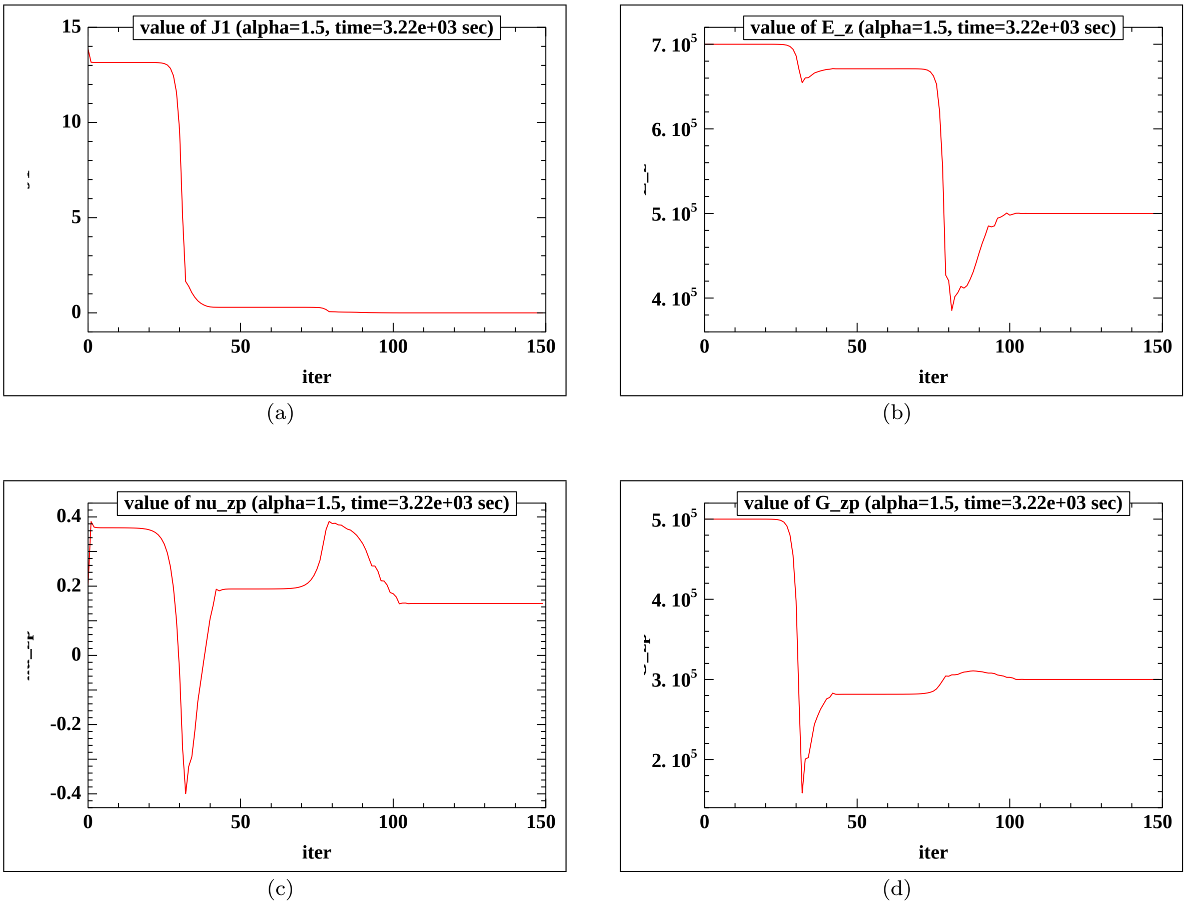

Figure 3: Convergence history for an academic example with target values $E_z = 5 \times 10^5$, $\nu_{zp} = 0.15$ and $G_{zp} = 3 \times 10^5$.

The numerical eigenvalue problem in tstep 1 is solved using the Arnoldi method. We use the implementation available in the PETSc/SLEPc open-source libraries. The discretization of this problem is done using a mesh with 182 elements and 1432 nodes (20-noded hexahedral elements), shown in Figure 2. The descent direction in step 3 is calculated using a quasi-Newton method, namely the BFGS method. This computation uses the gradient of J, calculated in step 2. The descent parameter $\gamma^{(k)}$ in step 4 of the algorithm is obtained through a line-search procedure ensuring that the strong Wolfe conditions are verified (see chapter 3 of [6]).

Figures 3(a)-3(d) show the convergence history for an academic example whose target values are $E_z = 0.5 \times 10^6, \nu_{zp} = 0.15$ and $G_{zp} = 0.3 \times 10^6$. The other two parameters $E_p = 1 \times 10^6$ and $\nu_p = 0.2$ are fixed during the optimization process. This means that the reference (*"measured"*) natural frequencies are produced numerically using these target values in the damaged zone Ω_D (in Figure 2 in red). In the rest of the domain (in Figure 2 in yelow) the material is isotropic with values $E = 1 \times 10^6$ and $\nu = 0.2$. The initial guess for (E_z, ν_{zp}, G_{zp}) is $(0.7 \times 10^6,\ 0.2,\ 0.5 \times 10^6)$. The figures show the convergence after 150 iterations of the functional J and the target values for material coefficients are recovered.

5. Conclusions and future work

We described a method that combines free material design with an inverse optimization problem in order to find the elastic coefficients that fit the measured modal parameters of the Cabril dam. The main ingredient of the method is the parametric derivative of the eigenvalues, which allows to use gradient type methods to solve the minimization problem described in section 3. We note that the functional J in consideration is non-linear and may have more than one minimum point.

In the future we intend to generalize this method in several ways. The most direct development is to use a general constitutive law (21 constants) instead of the transversely isotropic law. We could also allow the material coefficients to vary from element to element in the damaged zone in order to allow for different degrees of damage. Finally, to make the simulation more realistic, the effect of the water on the eigenvalues should be taken into account using for example H.M. Wastergaard's approach.

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